**OPTIMAL ORBITAL COVERAGE OF THEATER OPERATIONS AND TARGETS**

THESIS

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THESIS

Presented to the Faculty

Department of Aeronautics and Astronautics

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Astronautical Engineering

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March 2007

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## Abstract

The use of satellites as a tactical asset to support theater operations is a desired capability for future space operations. Unlike traditional satellite systems designed to provide coverage over the entire globe or large regions, tactical satellites would provide coverage over a small region which can be modeled as a single ground point defined by a latitude and longitude. In order to provide sufficient utility as a theater asset, a satellite should be placed in an orbit that provides a maximum amount of coverage of the target ground point. This study examined the optimization of orbit parameters to maximize the number of passes made over a target. An optimization algorithm was developed to maximize the number of passes made while also minimizing the distance from the satellite to the target. Single satellite coverage properties as well as two and three satellite constellations were analyzed.

## Acknowledgments

I would like to thank my faculty advisor, Lt Col Nathan Titus, for his guidance and support throughout the course of this thesis effort. I would also like to thank my fellow classmates whose advice and assistance was greatly appreciated.

Kimberly A. Sugrue

# Table of Contents

Page

Page

## List of Figures

Figure Page

## List of Tables

Table Page

OPTIMAL ORBITAL COVERAGE OF THEATER OPERATIONS AND TARGETS

# 1 Introduction

## 1.1 Background

### *1.1.1 Tactical Satellites and Responsive Space Operations*

Space assets have become an important and often critical part of military operations. Satellites are employed in a variety of missions including surveillance, communication, and navigation. Currently, satellite systems are managed as national assets but there is a strong interest in developing satellites that would be managed as a tactical asset. Using space assets as a tactical tool is part of a strategy termed responsive space operations. The concept of responsive space operations focuses on the ability to launch space assets in response to an emerging threat or identified need. A tactical satellite, launched in support of a planned or ongoing theater operation, would be under the control of the theater commander and make space capabilities a tactical asset. Tactical satellites could fill various missions such as providing additional surveillance of the theater or augmenting communications systems.

As stated by General Cartwright, tactical satellites must "demonstrate that operationally relevant, rapidly deployable spacecraft can support military operations anywhere on Earth." (2:3) In order to be rapidly deployable, tactical satellites will most likely be launched into low Earth orbits. This will allow the satellites to begin on-orbit operations in a minimal amount of time. In order to be operationally relevant, a satellite

will need to provide a sufficient amount of utility for its mission area. For surveillance, an important measure of a satellite's utility is the coverage it provides of the target area. In order to provide the most utility, a satellite should be placed in an orbit that maximizes the coverage of the theater target. Designing orbits that maximize the coverage of a specified target is an important research area for the concept of tactical satellites.

### *1.1.2 Orbit and Constellation Design*

Much of the work on orbit and constellation design has focused on the coverage of large areas. Constellations have been designed and optimized to provide coverage of the entire globe or the entire region enclosed by two bands of latitude. More recently, research has included coverage of a single ground point. Emery et al. explored the use of satellites in orbits whose inclination was matched to the value of the latitude of the target coverage area. Rendon demonstrated that an increase in the inclination above the target latitude tends to improve coverage performance. Wertz suggested an inclination 3 to 5 degrees above the target latitude for a 300 km orbit. The inclination, altitude, and longitude of ascending node of the orbit will affect the coverage performance of a satellite. Finding optimal combinations of these parameters will define orbits that will allow a satellite to provide the most utility in terms of coverage of a theater target.

## 1.2 Research Objectives

The primary objectives of this research were to find methods of optimizing the coverage of a theater target by a satellite and determining the optimum orbit parameters for the satellite. A theater target was specified by a latitude and longitude. Since the latitude of the target will vary, it was also desired that the effects of latitude location on

coverage optimization be observed.   The coverage of the target was measured as the number of daylight passes made over the target. Since tactical satellites might be employed as a single satellite or a small constellation, constellation design was also addressed.

### 1.3  Assumptions and Limitations

The use of satellites as a tactical asset imposes a practical limitation on the orbit altitude.  The research included only analysis of low-Earth orbits.  Since satellites designed for tactical use will need to be launched, placed in orbit, and operating in a timely manner, low Earth orbits are the most practical and suitable choice. Both circular and elliptical orbits were compared for performance in test cases, but only circular orbits were optimized and used for constellation design.

For the purposes of this research the primary mission of the satellite was assumed to be collection of visible imagery of a theater target.  Surveillance of a target may take various forms, but visible imagery is a common and highly valuable resource for theater operations.  Visible imaging requires that the target be illuminated by the sun, which limits a satellite to daytime operations.  Since this is an important constraint on the imaging opportunities, only passes made over the target during daylight were included in the coverage analysis for a satellite

The research focuses on optimizing coverage over a limited time period.  Since the satellites will be providing tactical support for theater operations, it was assumed the mission duration would be short. A nominal period of 30 days was chosen as the time period.  Since a satellite's longitude of ascending node regresses over time due to orbital

perturbations, the value selected for the node represents the initial value at the beginning of the 30 day period. The value is selected to allow the node to drift through its optimal value during the time period.

### 1.4 Methodology

In order to analyze the coverage properties of a satellite, an analytical approach was used to characterize the problem. A satellite's coverage of a target will be largely affected by its field of view which in turn is determined by its orbital altitude. The relationship between a satellite's field of view, orbital altitude, inclination, and the latitude of the target is examined analytically. The analysis is limited to circular orbits and several simplifying assumptions are made. In order to analyze the problem more accurately a numerical method was used.

In order to assess the coverage performance of a satellite, a computer program was designed to simulate the scenario. The simulation included an orbit propagator which used a numerical method to simulate the dynamics of a satellite and measure its position and velocity over the specified time period. The orbit propagation included the effects of the J2 perturbation caused by the oblateness of the Earth. The simulation propagated the position of the target site in inertial space by simulating the rotation of the Earth and propagated the position vector from the sun to the Earth. Using the simulated scenario, the program determined how many passes the satellite made with the target visible to the satellite and illuminated by the sun. The program measured the number of daylight passes the satellite made over the target, the length of each pass, the slant range from the satellite to the target during each pass, and the time at which passes occurred.

In order to optimize the performance of a satellite, the number of daylight passes made was chosen as the primary figure of merit. For a given orbital altitude and inclination, the longitude of ascending node was optimized to the value that provided the highest number of daylight passes. For a given altitude, the inclination which provides the highest number of daylight passes was determined and the associated trends examined. Constraints including a minimum elevation angle or a maximum slant range were also examined. The test cases at varying altitudes revealed a tradeoff between the number of daylight passes and the average slant range to the target. Since the slant range will affect the resolution of visible imagery it was added as another figure of merit.

The number of daylight passes and average slant range were selected as the performance criteria for orbit optimization. Maximizing the number of passes was chosen as one objective in order to provide the most opportunities for the satellite to capture imagery of the target. Minimizing the slant range to the target was chosen as the other objective in order to allow the highest resolution for the imagery. A multi-objective optimization algorithm using a weighted cost function was designed to select optimal orbits to meet the coverage objectives.

Using one of the optimized orbits, constellation design was examined. Two satellite constellations were designed by varying the mean anomaly of the second satellite and by varying the longitude of ascending node of the second satellite. Three satellite constellations were designed using the same techniques. A constellation was also designed for extended operations.

# 2. Literature Review

## 2.1 Introduction

Several related areas in the analysis of satellite coverage and orbit optimization have been examined. A large portion of research has focused on the design and optimization of satellite constellations. The number of satellites and orbit parameters chosen for each satellite are driven by coverage requirements. Research dedicated to tactical space applications has examined the types of orbits and constellations most suited to provide coverage for a theater or target. Orbit parameters are analyzed to provide the most utility in terms of coverage and to support tactical requirements.

## 2.2 Constellation Design

Much of the analysis of orbit optimization has focused on constellations of satellites with the goal of minimizing the number of satellites needed for given coverage requirements. Satellite coverage requirements vary depending on the application. For some applications, continuous coverage of the entire Earth's surface is desired. With continuous coverage, any given point on the Earth's surface is within view of a satellite at all times. In addition, some applications such as navigation may require continuous multiple satellite coverage. Such a system might require a user to be within view of a minimum of three satellites to operate. Other applications require continuous coverage a certain region often defined by a range of latitude bands. The region is often termed a zone and the zonal coverage required may be continuous or only periodic. For tactical satellites, the goal may only be to support or augment larger satellite systems. The

coverage provided by the tactical satellite will likely be intermittent and focused on a very small region or single target.

### *2.2.1 Continuous Global Coverage.*

Early research by Walker focuses on designing satellite constellations for continuous global coverage including multiple-satellite coverage, needed for applications such as navigation. Walker examines satellite coverage of the entire surface of the Earth to determine the constellation shape, orbital pattern and inclination, and minimum number of satellites which will meet a given coverage requirement. Walker's constellations consist of satellites in circular orbits of equal period to create uniform patterns in the constellation's coverage. The orbits of the constellation also share a common inclination because orbital perturbations dependant on inclination would make it difficult to maintain a stable relationship or pattern between orbits at varying non-zero inclinations. Using a combination of zero and high inclination orbits was also determined to be undesirable because of the practical difficulties of launching into different inclinations. (11: 559-560)

The Walker constellations, termed delta patterns, are made up of a total number of satellites (T) divided evenly into a number of orbital planes (P) with the satellites in each plane spaced evenly in mean anomaly. The orbital planes are all at the same inclination and spaced evenly by ascending node. The relative position of the satellites in one plane to the satellites in an adjacent plane is defined by a phasing parameter (F) and measured in "pattern units" where one pattern unit is equal to 360°/T. The phasing parameter is defined such that if a satellite in one plane is at its ascending node, there is a satellite in an adjacent plane which is at an orbital distance of F since passing its ascending node.

The phasing parameter is an integer in the range of 0 to (P-1). Thus the shape of a delta pattern can be described using the notation T/P/F and completely described by also including the inclination. The constellations designed by Walker to provide single continuous coverage of the Earth range in size from 5 to 40 satellites at an inclination of about 55 degrees. (11: 559-567)

Lang extends Walker's work to include large constellations of 20 to 100 satellites with the goal of examining low altitude constellations for continuous global coverage. Lang's work aims at examining constellations that might be used by smaller cheaper satellites and that are accessible with less expensive launch systems than higher altitude orbits. Symmetric constellations in non-polar orbits are examined in an attempt to optimize the constellation to have the minimum number of satellites required to meet a coverage requirement. Lang develops an algorithm to analyze the coverage of a constellation that is based on propagating the satellites over a set of test points spread over the Earth's surface. Lang maintains Walker's symmetric design and the same T/P/F and I (inclination) notation are used to describe the constellations. Lang found the optimal inclination varied between 55 to 75 degrees for continuous global coverage. Based on his results and comparison with previous results, Lang determines that for single continuous global coverage using 20 satellites or less, non-polar constellations are more efficient than polar constellations of the same size and that for an equal altitude, non-polar constellations could provide single continuous coverage with fewer satellites. For constellations of greater than twenty satellites, polar constellations proved more efficient. For multiple-satellite global coverage, the non-polar constellations are demonstrated to be more efficient. Lang also notes that polar constellations provide the

highest level of coverage in regions surrounding the poles while non-polar constellations typically provide highest levels of coverage in the mid or upper latitudes near the inclination of the orbits. Thus non-polar orbits may offer a more practical level of coverage since the Polar Regions are not usually the most important coverage area. (7: 1199-1207) Beste also examines constellations made up of satellites in polar orbits but expands the search to include non-symmetric constellations. Both continuous global coverage and continuous coverage from the pole to a given latitude are addressed. An equal number of satellites are in each polar orbit and the orbit planes are spaced in such a manner that the total number of satellites required is minimized. Beste compares the polar orbit method to a non-polar orbit method in which orbits are distributed in a uniform manner. Beste concludes that using a non-uniform distribution of orbital planes can reduce the required number of satellites by 10 to 20 percent. (1: 466-469)

***2.2.2 Continuous Zonal Coverage.***

Lang continues constellation analysis with optimization of continuous coverage of the mid-latitude region. Since the majority of the Earth's population and industrial activities are in the mid-latitudes, many satellite applications might require coverage of this region only instead of the entire globe. Lang again focuses on low-Earth orbits because of the interest in using small satellites and low cost launch methods. The mid-latitude region is defined as the area from 20 to 60 degrees of latitude (north and south). Both polar and non-polar inclinations are considered but constellations of polar orbits do not offer a significant savings in the number of satellites required to provide coverage of the mid-latitudes as opposed to the entire Earth's surface. Non-polar orbits set up in symmetric Walker constellations offer continuous coverage of the mid-latitudes with 2/3

to 3/4 the number of satellites that would be required for global continuous coverage for a given orbit altitude. For the case of a 400 nm (741 km) altitude orbit constellation, only 30 satellites are needed to cover the mid-latitudes while 40 are needed to cover the globe. Lang finds the optimal inclination for the constellations to typically range from 45 to 50 degrees depending on the selected altitude. (6: 595-598)

### *2.2.3 Minimizing Revisit Time.*

Lang also addresses satellite constellations which do not provide continuous coverage. Since not all applications may require continuous coverage of the Earth or a region and since low Earth orbits require a large number of satellites, constellations that have visibility gaps are examined. Rather than optimize a constellation to meet a given coverage requirement, a given number of satellites is optimized to determine the constellation which will provide the smallest maximum revisit time. The maximum revisit time is the longest period of time that a point on the ground will have no satellite coverage. Only circular orbits and symmetric constellations are considered and the T/P/F convention is used to describe the constellation. Lang develops an algorithm that takes in the total number of satellites as well as the coverage region which is defined as the area between two bands of latitude and can be as large as the entire globe. Since the constellations are symmetric, it is assumed that the maximum revisit time will be a function primarily of latitude with only a negligible dependence on longitude. In order to test the coverage of a constellation a set of ground points along a single meridian are used. The user chooses the number of ground points which are spaced evenly by latitude and the desired maximum revisit time for each ground point latitude. For each possible

combination of T/P/F the algorithm finds the optimal inclination to meet the user's criteria. (8: 1071-1086)

### *2.2.4 Small Satellite Constellations.*

Draim examines various constellation types and their applicability for small satellites. He addresses numerous tradeoffs that can be made to reduce cost including designing a constellation that uses the least number of satellites to meet a given level of coverage. He sees two main mission areas for small efficient satellites. The first is supporting existing larger satellite systems. The second is as a stand-alone system fulfilling various missions that must be performed at low altitudes. For low altitude constellations Draim considers only circular or near-circular orbits. Walker Delta Patterns and Beste's polar constellations are suggested for global coverage. For zonal coverage, Walker or Beste constellations can be reduced in size by using an inclination at or near the mean latitude of the zonal region. In this case satellites will appear over the zonal region several times in succeeding orbits. Draim also notes that since the orbits are circular and at an equal altitude and inclination, they will regress at the same rate. Other perturbing forces (i.e. sun, moon, uneven geoid) may require the satellites to have some station-keeping ability. (3: 1361-1365)

Draim suggests that small satellites at higher altitudes can be placed in elliptical orbits and optimized to provide the desired coverage. Draim has designed constellations at synchronous altitudes that provide continuous global coverage with a minimum number of satellites and with eccentricities that are slight to moderate. He has also designed constellations with 24 and 48 hour periods. Draim also considers a bi-level constellation which is essentially a constellation made up of two constellations each at a

different inclination. He provides an example of using three GEO satellites and augmenting the constellation with five satellites in circular orbits with eight-hour periods. (3: 1365-1368)

### 2.3 Tactical Space Applications

Increased interest on developing tactical space assets has led to the need to examine appropriate orbits and constellations for satellites that would provide support for theater operations. Tactical satellites could provide a theater commander with additional support for surveillance, communications, or other requirements and would allow the theater commander to have direct control over a space asset. In order to function practically as a tactical application, satellites need to be built and launched in a short amount of time and at a low cost. These requirements make small satellites and low Earth orbits the most applicable choice. Low Earth orbits will reduce launch costs and allow for smaller cheaper launch vehicles to be used. They are also more practical because there is less time required to reach the orbit than a higher altitude and the satellite can begin orbit checkout and operations as quickly as possible. Tactical satellites will provide coverage for a particular location where theater operations are planned to occur. While much research has been dedicated to the coverage of the Earth's surface or a zone of latitudes, theater coverage requires a single point or small region on the Earth's surface to be examined.

#### *2.3.1 Responsive orbits.*

Wertz evaluates several orbits for their use as responsive orbits. Responsive orbits would provide the means to have communications or high-resolution surveillance capabilities anywhere in the world within hours of an identified need. Responsive orbits would allow space assets to be used tactically instead of only strategically. Wertz identifies a Low Earth Orbit (LEO) Repeat Coverage Orbit as a potential responsive orbit. Wertz evaluates such an orbit to meet the needs of responsive missions including responsiveness, low cost, good coverage, and tactical applications. The LEO Repeat Coverage Orbit consists of a circular low Earth orbit, typically 300 km, at an inclination 3 to 5 degrees higher than the latitude of the target. This type of orbit could provide 3 to 5 minutes of coverage per orbit for 4 or 5 successive orbits. Small constellations of 3 to 4 satellites could be used to provide coverage every 90 minutes or 6 to 8 satellites to provide coverage every 45 minutes. Wertz suggests the LEO Repeat Coverage Orbit for visible, infrared, or radar surveillance. (12: 1-5)

#### *2.3.2 Partial Coverage Constellations.*

Hanson, et al develops a procedure to design partial coverage satellite constellations. The partial coverage constellations are designed to provide coverage of a single ground point and are not aimed at providing continuous coverage. The design algorithm seeks to minimize the number of satellites required for a constellation to meet the given coverage requirement. The coverage requirement is defined as a maximum gap time between coverage as well as a minimum elevation angle from the local horizontal of a ground point to the satellite. Only circular orbits are examined and satellite orbits share

a common inclination and altitude to avoid constellations that vary over time due to perturbations. Hanson et al. make a comparison between satellites in repeating orbits, orbits whose ground tracks repeat every day, and those in non-repeating orbits. Repeating orbits occur near altitudes of 500 km, 800 km, and 1200 km (and higher altitudes). Two scenarios are used to compare repeating versus non-repeating orbit performance. The desired ground point to be covered is located at 30 degrees latitude for the first scenario and 50 degrees for the second and a minimum elevation angle of 5 degrees is used in both cases. In both cases the minimum number of satellites is determined that will meet a maximum gap time constraint and then the minimum constellation inclination (above 28 degrees) which will still meet the gap time constraint is determined. Using the determined number of satellites and inclination, the longest gap time is minimized which may improve the maximum gap time to a shorter amount than the given constraint. The results show the repeating ground-track orbit constellations perform as well as and usually better than the non-repeating orbit constellations. The repeating orbit constellations also in some cases perform better and at a lower altitude than the non-repeating orbits. (5: 214-222)

### *2.3.4 Matched Inclination Constellations.*

Emery et al. examines the utility of using small satellite constellations to provide theater coverage. Only circular orbits are used with a nominal orbit altitude of 350 km and the orbits are inclined at the value of the latitude of the theater. The theater is an area surrounding a central target latitude and longitude and thus spans a range of latitude. Placing the inclination higher to match a northern latitude of the theater is shown to

improve the number of satellite passes made as opposed to placing the orbit inclination at the center latitude or in the southern portion of the theater. (4: 85-120)

The amount of coverage offered by a single satellite is examined as well as constellations of two, five, and eight satellites. Several different approaches to coverage optimization are used. For the case of a single satellite, the number of daylight passes made over a 30 day time period is maximized by optimizing the placement of the orbit's longitude of the ascending node. For the case of a two satellite constellation, three different approaches are used. The first approach is to maximize the number of daylight passes made over a thirty day time period by placing both satellites in an orbit sharing a common longitude of the ascending node and spacing the satellites evenly by mean anomaly. The second approach is to decrease the maximum out of view duration during a 30 day time period. The aim of the second approach is to spread coverage out throughout the day during the time period. The two satellites are placed in orbits with different longitudes of the ascending node. One satellite has a slightly higher value and the other has a slightly lower value than the longitude of the ascending node used in the single satellite optimization. The third approach is to prevent a gap in coverage by placing the satellites in orbits spaced evenly by longitude of the ascending node. One satellite is in an orbit with the same longitude of the ascending node as the single satellite case and the other satellite is in an orbit with a longitude of the ascending node 180 degrees from the first satellite's orbit. This approach is also used for a four satellite constellation. The satellites are spaced evenly with 90 degrees of separation in longitude of the ascending node. For the case of a five satellite constellation, two approaches are examined. The first approach is aimed at decreasing the maximum out of view time and

preventing gaps in coverage. Four satellites are spaced evenly by longitude of ascending node as described previously and then a fifth satellite is added at the same longitude of ascending node as one of the other satellites but shifted 180 degrees in mean anomaly. The second approach is to maximize the total number of daylight passes over a thirty day period by spacing the satellites by 50 degrees in longitude of the ascending node. For the eight satellite constellation, the approach is aimed at decreasing the maximum out of view duration and preventing gaps in coverage. Four satellites are spaced evenly by longitude of the ascending node and four additional satellites are placed in orbit with the same longitudes of the ascending node but spaced 180 degrees from the first satellites. (4: 168-179)

***2.3.5 Inclination and Altitude Effects on Target Coverage.***

Rendon explores the effects of orbital inclination and altitude on the target coverage provided by a satellite in low Earth orbit. The coverage over a particular ground point is examined for a thirty day time period by measuring the number of daylight passes made over the target. Several different cases are examined with the target at different latitudes. For each case, the effects of varying the inclination from the value of the target latitude are tested. At a target latitude of 33 degrees, the results show that increasing the inclination increases the number of passes made. The same trend is shown using a target latitude of 50 degrees. The effects of varying the orbit altitude are also shown by testing orbit altitudes of 100, 350, 600 and 800 km at each of the target latitudes. As the orbital altitude is increased the number of daylight passes increases due to the increasing size of the satellite's field of view. The inclination at which the

maximum number of passes occurs also is shown to increase with increasing orbit altitude. (9: 37:49)

# 3. Methods

## 3.1 Analytical analysis

The first approach used to examine the problem was an analytical analysis of the coverage properties of satellites. Satellites in circular orbits lend themselves well to analytical analysis because the coverage properties do not vary as the satellite travels around the Earth. Although an analytical approach provides insight, it also has limitations. Analytical approximations do not take into account orbital perturbations or other factors that will play an important role in the coverage properties of a real satellite.

### *3.1.1 Coverage Geometry*

The coverage geometry for a satellite is depicted in figure 3.1.

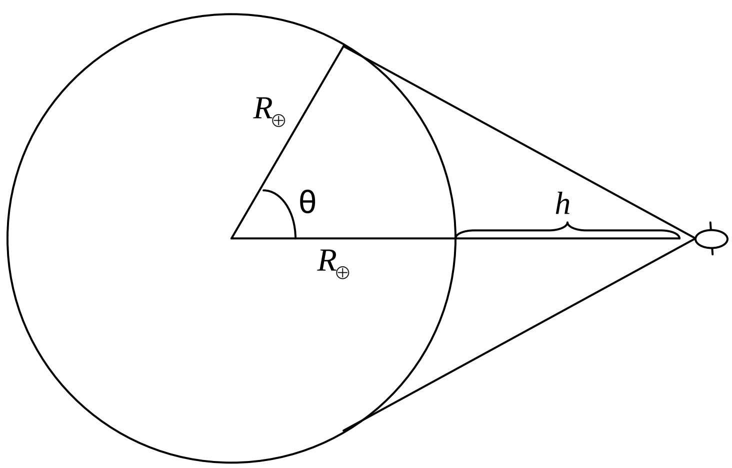


**Fig 3.1 Satellite Coverage Geometry**

A satellite's field of view on the Earth's surface will be a circle and the size of the field of view will be determined by the satellite's altitude. For a satellite in a circular orbit, the field of view will remain a constant size while a satellite in an elliptical orbit will have a

field of view whose size varies as the satellite's altitude changes. The Earth central angle ($\theta$) can be used to describe the size of half the satellite's field of view. For a satellite in a circular orbit and using a simplified spherical Earth model, the Earth central angle can be determined from equation (3.1). The equation can also be used to determine the Earth central angle for a point on an elliptical orbit with a particular altitude, *h*.

$$\cos\theta = \frac{R_\oplus}{R_\oplus + h} \tag{3.1}$$

The field of view shown in figure 3.1 is limited only by the satellite's altitude which determines where the local horizon is and thus the farthest point that is within the satellite's field of view. Many satellites have additional constraints on their field of view. There may be a minimum elevation angle required for the satellite to operate effectively due to obstructions in its line of sight. The satellite might have a maximum operating slant range for its onboard instruments. Figure 3.2 depicts an elevation angle (ε) constraint which creates an effective field of view limited further than the horizon.

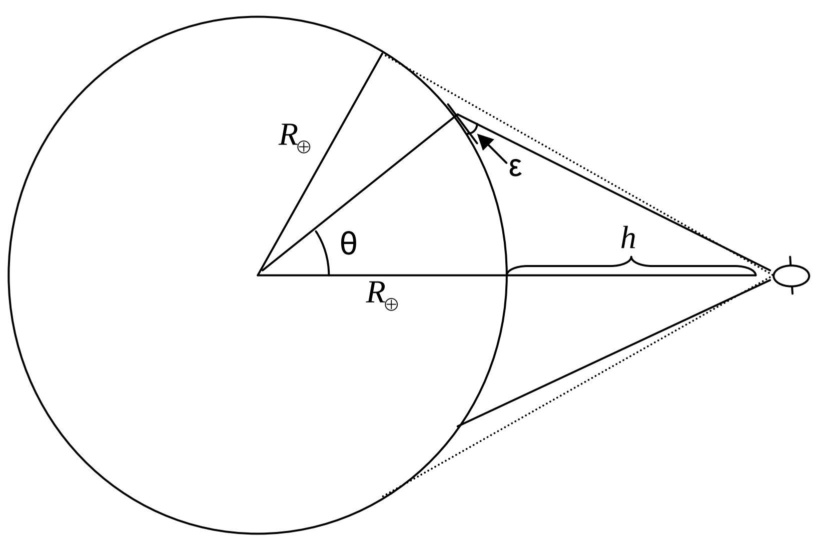


**Fig 3.2 Satellite Coverage Geometry with Minimum Elevation Angle**

If the field of view is constrained by an elevation angle requirement, the Earth central angle can be determined using equation (3.2).

$$\cos(\theta + \varepsilon) = \frac{R_{\oplus} \cos \varepsilon}{R_{\oplus} + h} \tag{3.2}$$

Figure 3.3 depicts the coverage for a satellite with a maximum slant range ( $\rho$ ).

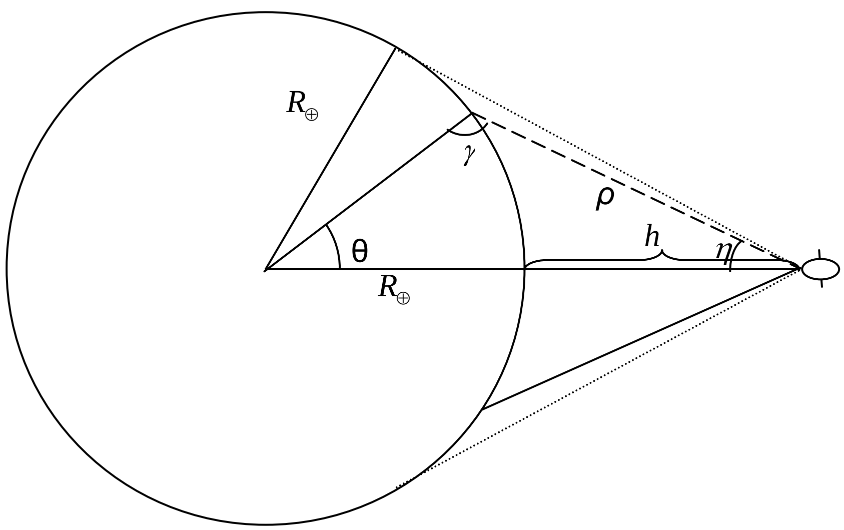


**Fig 3.3 Satellite Coverage Geometry with Maximum Slant Range**

If the field of view is constrained by a maximum slant range, the Earth central can be determined from equations (3.3) to (3.5). (10:781-782)

$$\cos \eta = \frac{\rho^2 + (R_{\oplus} + h)^2 - R_{\oplus}^2}{2\rho(R_{\oplus} + h)} \tag{3.3}$$

$$\sin \gamma = \frac{(R_{\oplus} + h)\sin \eta}{R_{\oplus}} \tag{3.4}$$

$$\theta = 180^{\circ} - (\eta + \gamma) \tag{3.5}$$

### *3.1.2 Latitude Coverage*

Using the geometry described for a satellite in a low Earth circular orbit over a spherical Earth, the satellite coverage of a particular circle of latitude can be approximated. For a given latitude, L, the coverage provided by a satellite in a circular low Earth orbit falls into one of three categories. If the orbit is at an inclination that is less than the Earth central angle there will be zero coverage of the latitude because the satellite's field of view will never reach the latitude. If the orbital inclination falls between (L-θ) and (L+θ) there will be one coverage period during each orbit pass. If the orbital inclination is greater than (L+θ), there will be two coverage periods during each pass, one as the satellite is ascending and one as it is descending.

For a circular orbit with Earth central angle, θ, and inclination, i, equations (3.6) and (3.7) can be used to analyze the coverage of a specific latitude, L: (13:170)

For L-θ < i ≤ L+θ:

$$\phi = \cos^{-1}\left(\frac{-\sin\theta + \cos i \sin L}{\sin i \cos L}\right) \tag{3.6}$$

For L+θ ≤ I ≤ 90:

$$\phi = \sin^{-1}\left(\frac{\sin\theta + \sin L \cos i}{\sin i \cos L}\right) + \sin^{-1}\left(\frac{\sin\theta - \sin L \cos i}{\sin i \cos L}\right) \tag{3.7}$$

These equations apply to a latitude in the northern hemisphere and a satellite in a direct orbit. The parameter $\phi$ represents half the coverage angle (measured in longitude) that the satellite provides in one pass. Thus a satellite will cover a region $2\phi$ degrees of longitude at latitude L during each pass. However equations (3.6) and (3.7) do not take into account the rotation of the Earth. Since the Earth is rotating as the satellite covers the region, the actual range of longitude covered will be smaller than $2\phi$.

Since equations (3.6) and (3.7) do not take into account the rotation of the Earth, a correction can be made to the longitude range, $2\phi$, using equations (3.8) and (3.9).

$$2\phi_{corrected} = 2\phi - \left(\frac{2\phi}{360} * P\right)\omega_{\otimes} \tag{3.8}$$

(note: $\phi$ in deg, Earth rotation rate in deg/s)

Where P is the period of the orbit given by

$$P = 2\pi\left(\frac{(R_{\oplus} + h)^3}{\mu}\right)^{1/2} \tag{3.9}$$

For a circular orbit at a given altitude, an examination of equations (3.6) and (3.7) show that if the target latitude, L, is greater than the Earth central angle, θ, the following is true: As the inclination is increased from (θ-L) to (θ+L) the longitude range $\phi$ will increase. The longitude range will reach its maximum at an inclination equal to (θ+L) and will continue to decrease after (θ+L) to 90 degrees. Figure 3.4 shows the general trend for various target latitudes. The orbit used was at 350 km altitude.

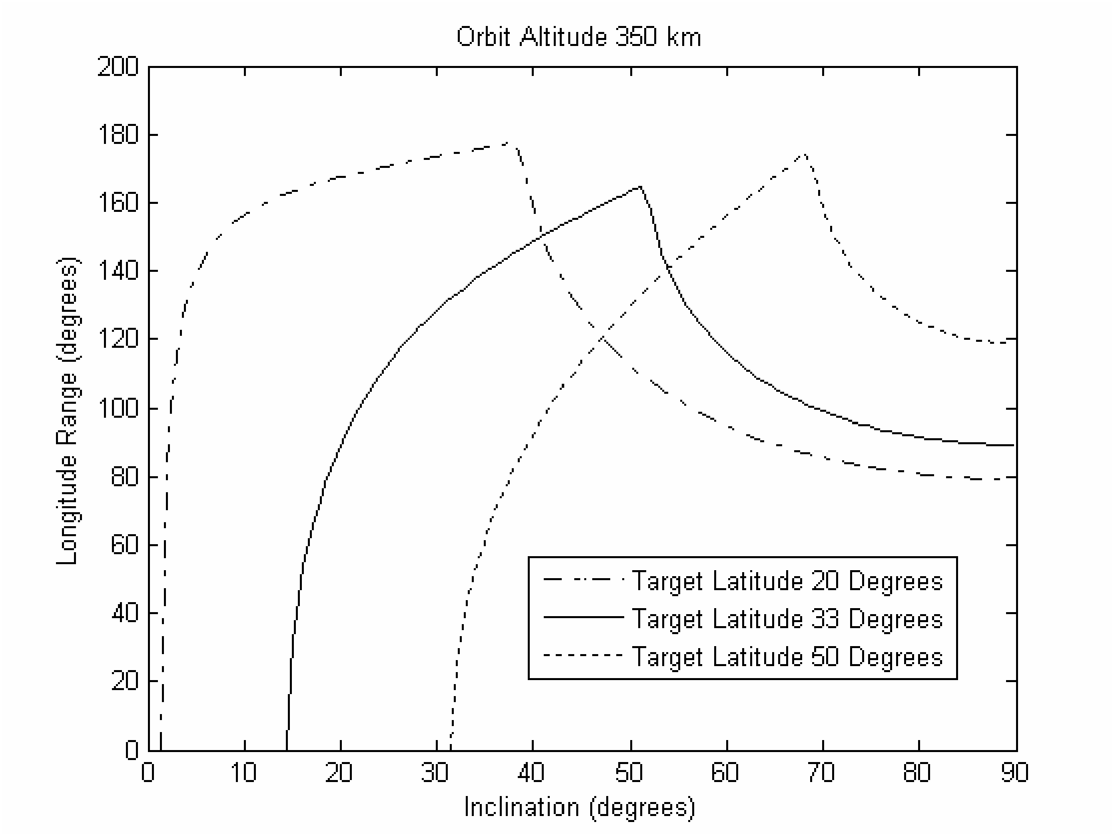


**Figure 3.4 Range of Coverage for Various Target Latitudes**

### *3.1.3 Target Coverage*

A satellite in low Earth orbit which seeks to cover a certain target latitude and longitude and which has an inclination near the target latitude ((θ-L) to (θ+L)) will follow a pattern in which it makes a series of successive passes during which it has coverage of the target followed by a number of passes during which it has no coverage of the target. This pattern will continue to repeat itself. The range of longitude that the satellite covers will affect the number of successive passes that have coverage of the target.

For a satellite in a given orbit with period, P, the ground track of the satellite will appear to shift westward in longitude with every orbit pass because the orbit is fixed in inertial space while the Earth is rotating eastward.

The shift ($s$) can be measured using equation (3.10).

$$s = P * \omega_{\oplus} \tag{3.10}$$

For an orbit with a longitude range $2\phi$, if $l_1$ represents the beginning of the longitude range (eastern most longitude) and $l_2 = l_1 + 2\phi$ represents the end (western most longitude) of the longitude range, then after one pass

$$\begin{aligned} l_1 &= l_1 - s \\ l_2 &= l_2 - s \end{aligned}$$

If the target longitude $l_t$ was exactly at $l_1$ for a given pass, then on the subsequent pass the target would be located at:

$$l_1 + s$$

And on the following pass:

$$l_1 + 2s$$

And so on until

$$l_1 + (x)s > l_2$$

At this point the target longitude is no longer within the longitude range and cannot be seen by the satellite.

Thus the number of passes in which the target is in view is directly related to

$$\frac{l_2 - l_1}{s} = \frac{2\phi}{s}$$

For the case described above the number of successive passes is given by

$$\#Passes = \left\langle \frac{2\phi}{s} + 1 \right\rangle \tag{3.11}$$

where $\langle \ \rangle$ denotes "integer portion of".

However this is only for the case where the target longitude was initially at $l_1$. If the target longitude is initially at $l_1 + \sigma$ (where $\sigma < s$) then on the following pass the target longitude would be located at

$$l_1 + s + \sigma$$

And on the next pass at

$$l_1 + 2s + \sigma$$

And so on until it reaches the point where

$$l_1 + (x)s + \sigma > l_2$$

The number of passes will be related to

$$\frac{l_2 - l_1 - \sigma}{s} = \frac{2\phi}{s} - \frac{\sigma}{s}$$

The number of passes will be given by

$$\# Passes = \left\langle \frac{2\phi}{s} - \frac{\sigma}{s} + 1 \right\rangle$$

Since $\frac{\sigma}{s} < 1$ the number of passes will only be reduced by 1 at the most. This means that the target will always be in view for $\left\langle \frac{2\phi}{s} \right\rangle$ to $\left\langle \frac{2\phi}{s} + 1 \right\rangle$ successive passes regardless of the initial longitude at which it comes into view.

A larger value of $\phi$ will mean a larger number of successive passes in which the target is within view of the satellite. However, since the number of passes will always be an integer, there will be a range of $\phi$ values (and hence a range of inclination values) which will result in the same number of passes. As an example, a 350 km altitude circular orbit will be considered. The target site will be at a latitude of 33 degrees and

longitude 44 degrees.  At an altitude of 350 km, a satellite will have an Earth central angle of approximately 18.56 degrees.  As shown earlier, the largest value of $\phi$ will occur at an inclination equal to the target latitude plus the Earth central angle.  In this case, that inclination would be 51.56 degrees.  The value of $2\phi$ at an inclination of 51.56 degrees, as calculated from equation (3.8), is approximately 165 degrees.  On each orbit the satellite will pass over a range of 165 degrees of longitude at latitude 33 degrees.  The actual longitude values the satellite cover will shift by 23 degrees on each orbit, as calculated from equation (3.10).  Once the target comes into view, the number of successive passes made over the target will range from 7 to 8 passes, as determined by equation (3.11).  A series of successive passes will then be made without the target in view and the process will repeat itself.  For this case, the process will take about a day so the number of successive passes is the number of passes made per day.  Table 3.1 shows a range of inclinations for this case and the resulting parameters.

**Table 3.1 Number of Passes Made per Day for Range of Inclination Values**

| Inclination (degrees) | $2\phi$ (degrees) | Number of Passes Made per Day |
|---|---|---|
| 33 | 135 | 6 |
| 35 | 140 | 7 |
| 37 | 144 | 7 |
| 39 | 147 | 7 |
| 41 | 150 | 7 |
| 43 | 153 | 7 |
| 45 | 156 | 7 |
| 47 | 159 | 7 |
| 49 | 162 | 8 |
| 51 | 165 | 8 |
| 51.56 | 165 | 8 |

As shown in the table, an inclination of 49, 51 or 51.56 degrees yields the highest number of passes per day. These results imply that in order to maximize the number of passes made over a target, the orbital inclination should be at or very close to the value of the Earth central angle plus the latitude of the target.

### 3.2 Computer Simulation

The analytical analysis applies to a simplified case and has various limitations. It does not take into account whether a target site is in daylight which is critical for visible imaging systems. It also does not include the effects of orbital perturbations and allows only for the examination of circular orbits. For further analysis a computer simulation was used. The simulation propagated a satellite over a thirty day time period and measured its coverage of a selected target site. The coverage was measured as the number of daylight passes made by the satellite while within view of the target site. The J2 orbital perturbation was simulated to include its orbital effects. Satellites in both circular and elliptical orbits were simulated as well as constellations of satellites.

#### *3.2.1 Satellite Propagation*

The satellite's position and velocity vectors were propagated using numerical integration. The two-body equations of motion for a satellite were used as well as a perturbing acceleration due to the oblateness of the Earth (J2). The state vector and differential equations of motion are shown in equations (3.12) and (3.13).

$$\begin{Bmatrix} \vec{r} \\ \vec{v} \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} \tag{3.12}$$

$$\frac{d}{dt}\begin{Bmatrix}\vec{r}\\ \vec{v}\end{Bmatrix}=\begin{Bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ -\frac{\mu x}{r^3}+\frac{3\mu R_\oplus^2 J_2 x}{2r^5}\left(\frac{5z^2}{r^2}-1\right)\\ -\frac{\mu y}{r^3}+\frac{3\mu R_\oplus^2 J_2 y}{2r^5}\left(\frac{5z^2}{r^2}-1\right)\\ -\frac{\mu z}{r^3}+\frac{3\mu R_\oplus^2 J_2 z}{2r^5}\left(\frac{5z^2}{r^2}-3\right)\end{Bmatrix} \tag{3.13}$$

The following quantities were used:

$J_2 = 0.001082$
$R_\oplus = 6378.1363$ km
$\mu = 398600.4415$ km$^3$/s$^2$

### *3.2.2 Sun position vector*

In order to determine if the target site was in daylight, the position vector from the Earth to the sun was needed. The position vector from the Earth to the sun ($\vec{r}_{\odot}$) in an Earth-centered inertial frame was determined using the following equations. (10:265-268) The Julian date ($JD_{\mathrm{UT1}}$) was converted to time in Julian centuries since epoch ($T_{\mathrm{UT1}}$) using equation (3.14).

$$T_{\mathrm{UT1}} = \frac{JD_{\mathrm{UT1}} - 2451545}{36525} \tag{3.14}$$

The mean longitude of the sun ($\lambda_{M_{\odot}}$) was determined using equation (3.15).

$$\lambda_{M_{\odot}} = 280.460^\circ + 36000.770 T_{\mathrm{UT1}} \tag{3.15}$$

The mean anomaly of the sun ( $M_{\odot}$ ) was estimated using equation (3.16).

$$M_{\odot} = 357.5277233^{\circ} + 35999.05034 T_{\mathrm{UT1}} \tag{3.16}$$

The ecliptic longitude ( $\lambda_{ecliptic}$ ) was determined using equation (3.17).

$$\lambda_{ecliptic} = \lambda_{M_{\odot}} + 1.914666471^{\circ} \sin(M_{\odot}) + .019994643 \sin(2M_{\odot}) \tag{3.17}$$

The magnitude of the position vector to the sun was solved for using equation (3.18).

$$r_{\odot} = 1.000140612 - .016708617 \cos(M_{\odot}) - .000139589 \cos(2M_{\odot}) \tag{3.18}$$

The obliquity of the ecliptic ( $\varepsilon$ ) was determined using equation (3.19).

$$\varepsilon = 23.439291^{\circ} - .0130042 T_{\mathrm{UT1}} \tag{3.19}$$

The sun's position vector was calculated using equation (3.20).

$$\vec{r}_{\odot} = 1.4959787066 * 10^{8} \begin{bmatrix} r_{\odot} \cos(\lambda_{ecliptic}) \\ r_{\odot} \cos(\varepsilon) \sin(\lambda_{ecliptic}) \\ r_{\odot} \sin(\varepsilon) \sin(\lambda_{ecliptic}) \end{bmatrix} \tag{3.20}$$

### ***3.2.3 Site Position vector***

In order to determine if the target site was within view of the satellite, the target site's position vector was required. The target site's latitude ( $\theta_{site}$ ), longitude ( $\phi_{site}$ ), and altitude ( $h_{site}$ ) were used to determine its position vector in an Earth-centered Earth-fixed coordinated frame using the semi-major axis ( $a_{\oplus}$ ) and eccentricity ( $e_{\oplus}$ ) of the Earth as shown in equation (3.21).

$$\vec{r}_{site_{ECEF}} = \begin{bmatrix} \left( \dfrac{a_\oplus}{\sqrt{1-e_\oplus \sin^2(\theta_{site})}} + h_{site} \right) \cos(\theta_{site}) \cos(\phi_{site}) \\ \left( \dfrac{a_\oplus}{\sqrt{1-e_\oplus \sin^2(\theta_{site})}} + h_{site} \right) \cos(\theta_{site}) \sin(\phi_{site}) \\ \left( \dfrac{a_\oplus}{\sqrt{1-e_\oplus \sin^2(\theta_{site})}} (1-e_\oplus) + h_{site} \right) \sin(\theta_{site}) \end{bmatrix} \tag{3.21}$$

The position vector of the target site was determined in an Earth-centered inertial frame using the following approach: (10:189-191)

The Julian date was converted to Julian centuries since epoch ($T_{\mathrm{UT1}}$) using equation (3.14). The Greenwich mean sidereal time at midnight ($\theta_{GMST_0}$) was calculated using equation (3.22).

$$\theta_{GMST_0} = 100.4606184^\circ + 36000.77005361 T_{\mathrm{UT1}} + .00038793 T_{\mathrm{UT1}}^2 - 2.6*10^{-8} T_{\mathrm{UT1}}^3 \tag{3.22}$$

The Greenwich mean sidereal time ($\theta_{GMST}$) at $t$ seconds after midnight was determined using equation (3.23).

$$\theta_{GMST} = \theta_{GMST_0} + \omega_\oplus t \tag{3.23}$$

The local sidereal time ($\theta_{LST}$) was calculated using equation (3.24).

$$\theta_{LST} = \theta_{GMST} + \phi_{site} \tag{3.24}$$

The Earth-centered inertial position vector was determined using equation (3.25).

$$\vec{r}_{site} = \begin{bmatrix} \cos(-\theta_{LST}) & \sin(-\theta_{LST}) & 0 \\ -\sin(-\theta_{LST}) & \cos(-\theta_{LST}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{r}_{site_{ECEF}} \tag{3.25}$$

### *3.2.4 Site Illumination*

To determine whether the target site was illuminated by the sun, the angle ($\theta_{\odot/site}$) between the sun's position vector and the site's position vector was calculated as shown in equation (3.26).

$$\theta_{\odot/site} = \cos^{-1}\left(\frac{\vec{r}_{site} \bullet \vec{r}_{\odot}}{r_{site} r_{\odot}}\right) \tag{3.26}$$

If the angle between the sun and site position vectors was between 0 and 90 degrees, the site was considered to be illuminated by the sun.

### *3.2.5 Slant Range*

The distance from the satellite to the target ($\vec{\rho}$) was measured using equation (3.27).

$$\vec{\rho} = \vec{r}_{sat} - \vec{r}_{site} \tag{3.27}$$

### *3.2.6 Site Visibility*

The following methods were used to determine whether the target site was within view of the satellite:

The position vector ($\vec{\rho}$) from the target site to the satellite was calculated using equation (3.27). The elevation angle ($\varepsilon$) from the site to the satellite was determined from equation (3.28).

$$\varepsilon = \frac{\pi}{2} - \cos^{-1}\left(\frac{\vec{\rho} \bullet \vec{r}_{site}}{\rho r_{site}}\right) \tag{3.28}$$

If the elevation angle was greater than the minimum required elevation angle (or zero if no minimum elevation angle had been designated) then the site was considered to be visible to the satellite.

### *3.2.7 Simulation Parameters*

For each simulation the same initial epoch date and time of 1 June 2004 0000Z was used. The date and time were kept the same to allow for comparisons to be drawn between different simulations with varying orbital parameters. During each simulation the satellite position and velocity vectors, the sun's position vector, and the target site position vector were propagated over a thirty day time period. The program checked whether the target site was illuminated by the sun and visible to the satellite using one minute (60 second) time steps. If the site was both illuminated and visible to the target, the satellite was determined to be making a daylight pass over the target. The duration of each daylight pass was measured as well as the slant range from the satellite to the target during each pass. The slant range was assumed to be the shortest slant range that occurred during the pass. The total number of passes and total coverage time for the 30 day period was determined as well as the average slant range and the maximum slant range to the target site.

### *3.2.8 Circular Orbits*

#### *3.2.8.1 Longitude of the Ascending Node Optimization*

For circular orbits, a numerical search was used to optimize the longitude of the ascending node for a simulation. The simulation was run at 36 degree increments in longitude of ascending node. The node which yielded the highest number of passes was selected and the simulation was run at 8 degree increments of longitude of ascending node from 36 degrees less than the selected value to 36 degrees greater than the selected

value. The node which yielded the highest number of passes was selected as the optimal longitude of the ascending node for that simulation.

#### *3.2.8.2 Optimum Inclination for Maximum Number of Daylight Passes*

For a circular orbit at a given altitude, it was desired to determine the inclination which would yield the highest number of daylight passes. The orbit was tested at varying inclinations to determine where the maximum number of daylight passes occurred. Tests were conducted at various target latitudes to show the effect on where the optimum inclination occurs. Various altitudes were also tested to see if trends remained the same as altitude was changed. Table 3.2 summarizes the test cases examined. The altitudes selected were low-Earth orbits.

**Table 3.2 Test Cases for Optimal Inclination Determination**

| Target Latitude | Circular Orbit Altitudes (km) |
|---|---|
| 33 degrees | 200, 300, 350, 400, 500, 600, 700, 800 |
| 0 degrees | 350 km, 800 km |
| 10 degrees | 350 km, 800 km |

#### *3.2.8.3 Optimum Inclination for Maximum number of Daylight passes with constrained slant range*

It was also desired to investigate inclination optimization when constrained by a maximum slant range. For this case a limit was set on the allowable slant range from the satellite to the target. The constraint on slant range was investigated because imaging equipment onboard a satellite might have a limiting altitude at which it can effectively

operate. If the slant range was greater than the constraint, the target was not considered to be visible to the satellite.

*Optimum Inclination for maximum number of daylight passes with constrained elevation angle*

In this case a minimum elevation angle greater than zero was used as a constraint. In many real world applications, a satellite requires an elevation angle greater than zero to effectively operate on a target. If the elevation angle from the target to the satellite was not greater than or equal to the designated minimum elevation angle, the target was not considered visible to the satellite.

### 3.2.9 Elliptical Orbits

Elliptical orbits were investigated to determine if they offered better performance than circular orbits. In order to make a comparison between circular and elliptical orbits, the semi-major axis was kept constant for the orbits being compared. For elliptical orbits a numerical search over longitude of ascending node and argument of perigee was used to find the optimum argument of perigee and longitude of ascending node. The combination of longitude of ascending node and argument of perigee that yielded the highest number of daylight passes was selected as the optimum values for those parameters.

### 3.2.10 Optimization Algorithm

The number of daylight passes made over the target and the average slant range from the satellite to the target are two important coverage properties. The maximum number of daylight passes will provide the maximum number of opportunities for imaging of the target. The average slant range will affect the resolution of the imagery and the minimum slant range distance will provide the highest resolution imagery. An orbit which provides a high number of daylight passes usually also has a high average slant range. In order to balance the tradeoffs between the number of passes and slant range, an optimization algorithm was developed and implemented as a computer program.

*3.2.10.1 Input*

The algorithm takes in a series of inputs as shown in table 3.3.

**Table 3.3 Optimization Algorithm Input Parameters**

| Target Parameters | Search Span | Time Parameters | Weighting Parameters |
|---|---|---|---|
| Target Latitude | Maximum Orbit Altitude | Time span | Maximum Passes Weight |
| Target Longitude | Minimum Orbit Altitude | Start Date | Minimum Slant Range Weight |
| Target Altitude | | | |
| Minimum Elevation Angle | | | |

The maximum and minimum altitudes define the span of orbit altitude that the algorithm will search over to select an optimum orbit. The times span specifies the length of time (in days) that the simulation will optimize over for an orbit and the start date specifies the

date on which the time period will start. The algorithm only examines circular orbits and is intended only for low Earth orbits. It is also assumed that the target latitude is greater than the Earth central angle of each altitude in the search span. The weighting parameters are used to indicate the importance that should be given to the number of daylight passes made and to the average slant range.

#### *3.2.10.2 Solution Space*

The solution space for the optimization problem is found by determining the approximate maximum number of daylight passes and corresponding average slant range as well as the approximate minimum average slant range and corresponding number of daylight passes. In order to find the minimum average slant range, the minimum orbit altitude as specified by the input parameter is used. The program simulates a satellite at the minimum orbit altitude and an inclination equal to the target latitude. The longitude of ascending node is optimized to the value that provides the maximum number of daylight passes. The average slant range is calculated as well as the number of daylight passes made. The program then increments the inclination value above the target latitude and again simulates the satellite and measures the average slant range and number of daylight passes made. The program continues to increment the inclination until a local minimum value for average slant range is determined for the minimum orbit altitude. This local minimum is considered the minimum value for average slant range ($range_{\min}$) for the solution space and the corresponding number of daylight passes is considered the minimum number of daylight passes ($pass_{\min}$) for the solution space.

The maximum number of daylight passes is determined by using the maximum orbit altitude. The program calculates the approximate inclination ($i_{\max}$) at which the maximum number of daylight passes will be made using equations (3.29) to (3.30).

$$\cos(\theta+\varepsilon)=\frac{R_{\oplus}\cos\varepsilon}{R_{\oplus}+h} \tag{3.29}$$

where h is the maximum orbit altitude and $\varepsilon$ is the minimum elevation angle.

$$i_{\max}=l_{target}+\theta \tag{3.30}$$

where $l_{target}$ is the latitude of the target.

The program simulates a satellite in an orbit at the maximum orbit altitude and calculated inclination value ($i_{\max}$). The longitude of the ascending node is optimized to provide the highest number of daylight passes. The number of daylight passes made are measured as well as the average slant range. The program then increments the inclination below the approximated value and simulates a satellite at the maximum orbit altitude and the new inclination value. This process is repeated until a local maximum value is found for the number of daylight passes. This local maximum is considered the maximum number of passes ($pass_{\max}$) for the solution space and the corresponding average slant range is considered the maximum slant range ($range_{\max}$) for the solution space.

The maximum and minimum bounds of the solution space are used to determine the span of passes and span of slant range as shown in equations (3.31) and (3.32).

$$span_{pass}=pass_{\max}-pass_{\min} \tag{3.31}$$

$$span_{range}=range_{\max}-range_{\min} \tag{3.32}$$

The spans of average slant range and number of passes are then used to find a scaling parameter so that changes in range can be compared with changes in the number of passes. Equation (3.33) is used to determine the scaling parameter ( $\sigma$ ) for average slant range. The scaling parameter is used to normalize the slant range and number of passes.

$$\sigma = \frac{span_{range}}{\left(\frac{span_{pass}}{\delta}\right)} \tag{3.33}$$

where $\delta$ represents the accuracy of the number of passes as determined by a simulation and is set at a default value of 5. The $\delta$ parameter is used to set a significance level for the number of passes. An increase of a single pass may not truly represent a better coverage property but could be the result of where the simulation stopped so a minimum of five passes is used to ensure that the difference in coverage is significant.

*3.2.10.3 Optimization Solution Method*

The optimization algorithm has two objectives, to maximize the number of daylight passes and to minimize the average slant range. To satisfy both objectives, a weighted cost function is used to find an optimal solution. A weighting parameter for the number of passes and a weighting parameter for the average slant range are used to determine the importance of each objective. By changing the weighting parameters, different solutions can be found. Equation (3.34) shows the cost function for the algorithm.

$$C = \sum_{i=1}^{30} \lambda_1 \frac{x_1}{\delta} - \lambda_2 \frac{x_2}{\sigma} \qquad (3.34)$$

where

$\lambda_1$ = Weighting parameter for number of passes
$\lambda_2$ = Weighting parameter for averge slant range
$x_1$ = Number of Daylight Passes
$x_2$ = Average Slant Range

The optimum orbit is found by maximizing the cost function within the constraints placed on orbit altitude. The parameter space is searched in orbital altitude increments of 100 km and at each altitude the inclination is incremented by 2 degrees from the target latitude up to the value of the earth central angle plus the target latitude. The altitude which yielded the maximum cost function is then selected to narrow the parameter space. The altitude range of 200 km around the selected altitude is searched at 50 km increments and again at each altitude the inclination is incremented by 2 degrees from the target latitude to the value of the earth central angle plus the target latitude. The altitude and inclination that yield the maximum cost function are selected as the optimum solution. When the optimum solution is found, the program provides the orbit parameters and coverage properties for the orbit selected as shown in table 3.4.

**Table 3.4 Optimization Algorithm Output**

| **Orbit Parameters** | **Orbit Coverage Properties** |
|---|---|
| Altitude | Number of Daylight Passes |
| Inclination | Total Coverage Time |
| Longitude of Ascending Node | Average Pass Length |
| | Average Slant Range |
| | Maximum Slant Range |

### 3.2.11 Constellation Design

The optimization algorithm is designed to optimize an orbit for a single satellite. If more than one satellite will be used, a constellation must be designed. In order to examine the performance of constellation designs, an orbit selected by the optimization algorithm was chosen. The orbit parameters chosen were an orbit altitude of 500 km and an inclination of 45 degrees which correspond with an equal weighting for the number of passes and the average slant range. Each satellite in the constellation was kept at the same orbital altitude and inclination. Constellations for two and three satellites were designed. Attention was also given to constellations intended for extended operations.

# 4 Results

## 4.1 Introduction

The test cases presented reveal trends associated with orbit coverage properties. A key result of the test cases is the tradeoff between the number of satellite passes made and the slant range from the satellite to the target. Since both of these properties are important for high resolution imagery, an optimization algorithm was used which takes into account and weights the objectives of maximizing the number of passes and minimizing the slant range. Using one of the optimized orbits, an examination of constellation properties was performed to design configurations for constellations of two and three satellites.

## 4.2 Optimum Inclination for Maximum number of Daylight passes

Various test cases were examined to determine the inclination that maximizes the number of daylight passes made. For visible imaging satellites, an orbit that provides the most opportunities to capture imagery of the target is desired. The average slant range to the target was also measured since it is another important consideration for target coverage. The test cases include varied orbit altitude, target latitude, slant range constraints, and elevation angle constraints.

### *4.2.1 Analytical Predictions*

The analytical analysis showed a direct correlation between the inclination of a satellite's orbit and the swath of longitude at a given latitude value that would be viewed each time the satellite completed an orbit around the Earth. An inclination at the value of

the satellite's Earth central angle plus the target latitude provided the largest swath of longitude. In addition a larger longitude swath was shown to correspond to an increased number of passes over a target latitude and longitude. These results imply that the maximum number of passes will be made at an inclination equal to the Earth central angle plus the value of the target latitude. To test this prediction, various orbit cases were used in the computer simulation.

#### *4.2.2 350 km Altitude Circular Orbit, Target Latitude 33 Degrees*

In order to determine the optimum inclination for a satellite in a 350 km circular orbit, a range of inclinations was tested to see where the maximum number of daylight passes occurs. The tests were run using a 30 day time period and the total number of daylight passes measured. For each pass counted, the target site was illuminated by the sun and therefore in daylight and the target site was visible to the satellite. The target site was considered visible if the elevation angle was greater than or equal to zero degrees. For each inclination tested, the longitude of the ascending node was optimized to yield the highest number of daylight passes. Figure 4.1 shows the results for a target at a latitude of 33 degrees. As expected, the number of daylight passes varies depending on the inclination. The number of passes increases as inclination is increased until it reaches its maximum at 51 degrees. This optimum inclination provided 186 passes during the 30 day time period. The number of passes then drops off steeply as the inclination is increased above 51 degrees. The optimum inclination occurs at 18 degrees above the target latitude and the Earth-central angle for an orbital altitude of 350 km is 18.56 degrees. Thus the analytical prediction that the optimum inclination occurs at the target latitude is consistent with the results.

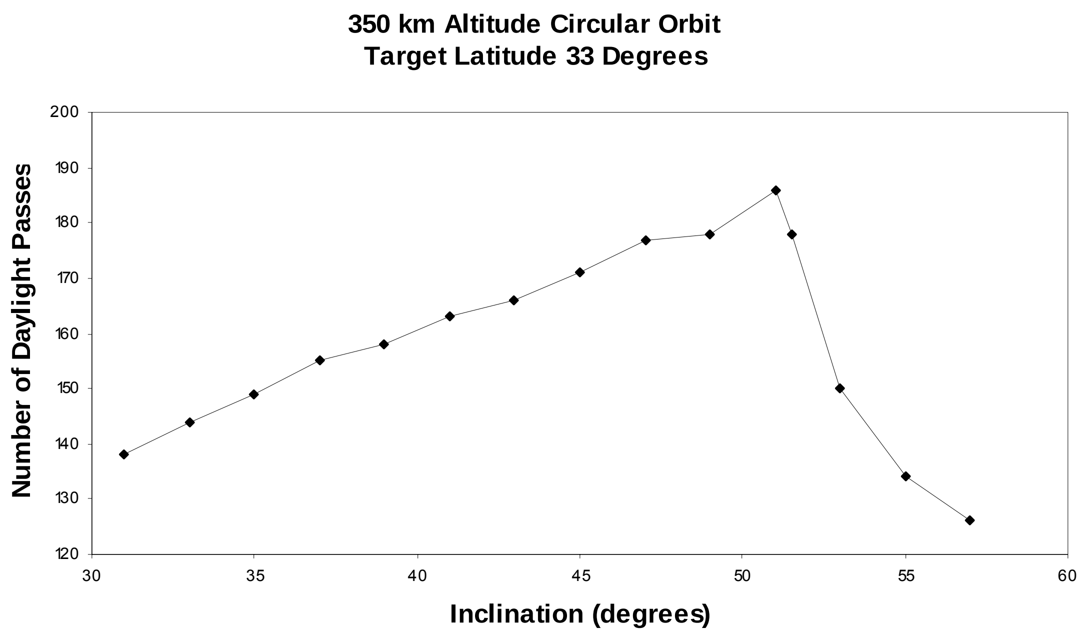


**Fig 4.1 Number of daylight passes made during a 30 day time span vs. inclination 350 km altitude circular orbit, target latitude 33°**

Table 4.1 provides additional parameters for the satellite coverage at the optimal inclination.

**Table 4.1 Summary of Coverage Properties**
**350 km Altitude Circular Orbit, 51° Inclination, Target Latitude 33°**

| | |
|---|---|
| Number of Daylight Passes | 186 |
| Total Coverage Time (hours) | 21.1 |
| Average Pass Length (minutes) | 6.8 |
| Average Slant Range to Target (km) | 1381 |
| Maximum Slant Range to Target (km) | 2102 |

The slant range, measured as the distance from the satellite to target, may play an important role in choosing an orbit. Although the inclination has been optimized for the

maximum number of passes, it may be undesirable in some cases to have a large slant range. The surveillance equipment carried on board a satellite may have a limitation on the distance at which it can function.

Figure 4.2 shows the slant range as a function of inclination for a 350 km altitude circular orbit and a target latitude of 33 degrees. As the inclination is increased above the target latitude, the slant range increases as well and the optimum inclination for maximum number of passes is also where the maximum average slant range occurs. This tradeoff between increased passes and increased slant range is an important consideration for orbit selection.

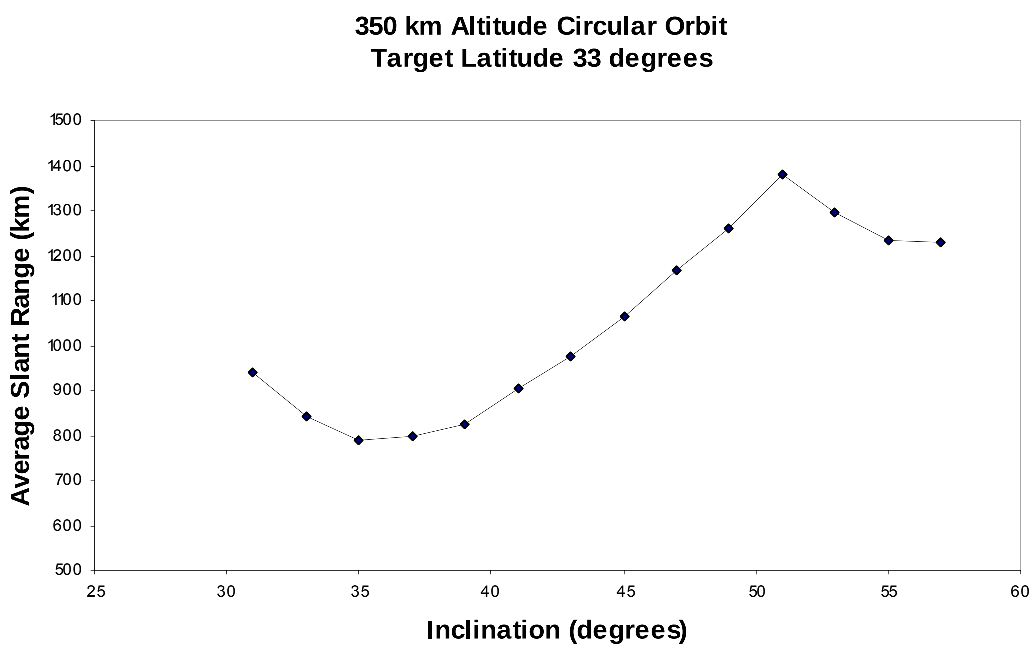


**Fig 4.2 Average slant range for 350 km altitude circular orbit and target latitude 33°**

### *4.2.3 800 km Altitude Circular Orbit, Target Latitude 33 Degrees*

The second case tested was an 800 km altitude circular orbit. The target latitude was kept at 33 degrees and a range of inclinations was tested to determine the inclination at which the maximum number of daylight passes occurs. The minimum elevation angle used was zero degrees and the time span 30 days. For each inclination tested the longitude of the ascending node was optimized to the value that yielded the highest number of daylight passes. Figure 4.3 shows the number of daylight passes for the values of inclination tested.

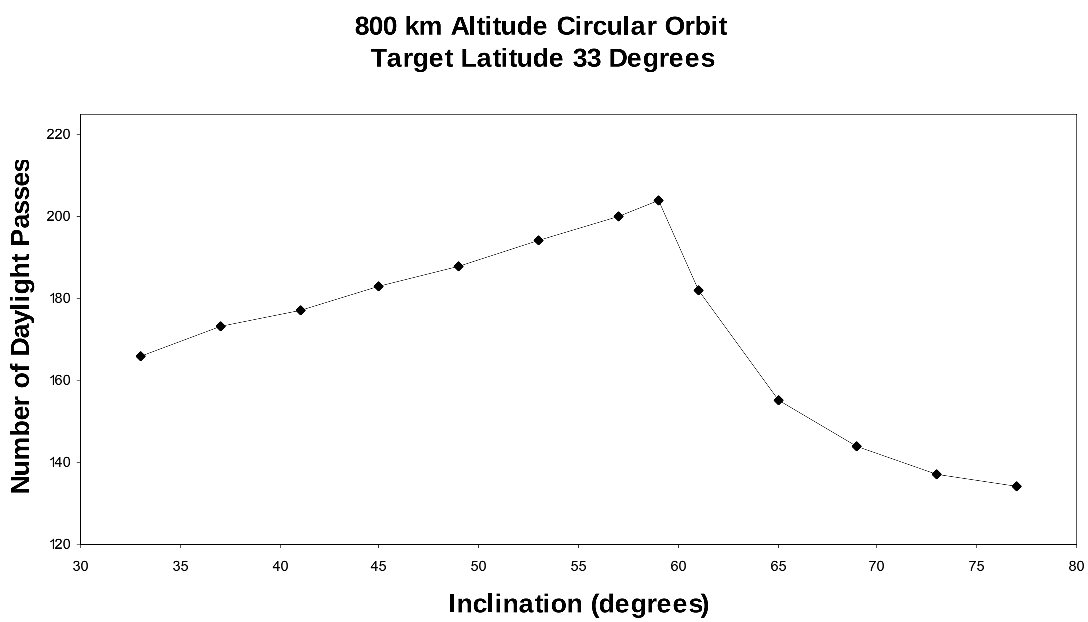


**Fig 4.3. Number of daylight passes made during a 30 day time span vs. inclination 800 km altitude circular orbit, target latitude 33°**

The maximum number of daylight passes made over the 30 day period is 204 passes which occurs at an inclination of 59 degrees. The trends shown are consistent with the

350 km altitude orbit. The number of passes increases as the inclination in increased above the target latitude until it reaches a maximum at 59 degrees. As the inclination is increased above 59 degrees the number of passes decreases quickly. The Earth-central angle for an 800 km altitude orbit is 27.31 degrees and the optimum inclination is 26 degrees above the target latitude. In comparison with the 350 km altitude orbit case, an increased number of daylight passes are made by the 800 km altitude case. This is an expected result since increasing the altitude of an orbit increases the satellite's field of view on the surface of the Earth.

Table 4.2 provides additional details about the satellite coverage at the optimum inclination of 59 degrees.

**Table 4.2 Summary of Coverage Properties**
**800 km Altitude Circular Orbit, 59° Inclination, Target Latitude 33°**

| | |
|---|---|
| Number of Daylight Passes | 204 |
| Total Coverage Time (hours) | 39.2 |
| Average Pass Length (minutes) | 11.5 |
| Average Slant Range to Target (km) | 2101 |
| Maximum Slant Range to Target (km) | 3274 |

The total coverage time and average pass length have increased in comparison to the 350 km case, but the average and maximum slant range have also increased. The average slant range versus inclination for the 800 km case is shown in figure 4.4.

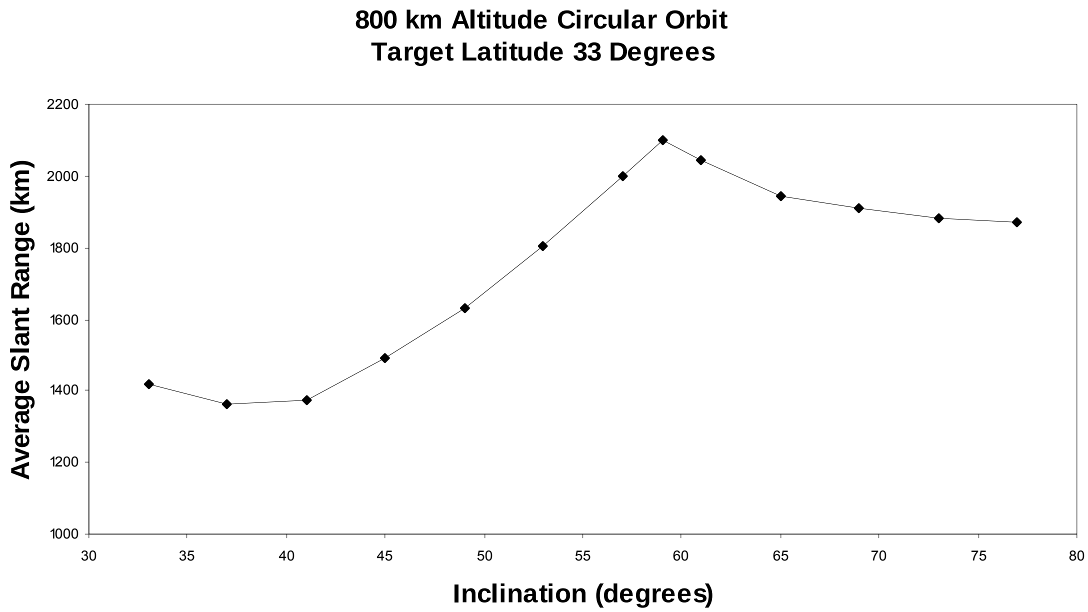


**Fig 4.4 Average slant range for 800 km altitude circular orbit and target latitude 33°**

The average slant range is at its maximum at the optimum inclination for maximum number of daylight passes. The trend is consistent with the trend seen for the 350 km altitude orbit case.

### *4.2.4 Varying Altitude Comparison*

Table 4.3 compares the optimum inclination for maximum number of daylight passes for various orbit altitudes.

**Table 4.3 Comparison of Coverage Properties for Varying Altitudes Target Latitude 33°**

| Orbital Altitude (km) | 200 | 300 | 400 | 500 | 600 | 700 | 800 |
|---|---|---|---|---|---|---|---|
| **Optimal Inclination** | 46 | 49 | 51 | 53 | 56 | 58 | 59 |
| **Earth Central Angle +Target Latitude** | 47.2 | 50.2 | 52.8 | 55 | 56.9 | 58.7 | 60.3 |
| **Number of Daylight Passes** | 172 | 178 | 186 | 198 | 198 | 202 | 204 |
| **Total Coverage Time (hours)** | 15.1 | 19.3 | 24.3 | 29.2 | 31.4 | 34.7 | 39.2 |
| **Average Pass Length (minutes)** | 5.3 | 6.5 | 7.8 | 8.8 | 9.5 | 10.3 | 11.5 |
| **Average Slant Range (km)** | 1007 | 1231 | 1410 | 1603 | 1805 | 1974 | 2101 |
| **Maximum Slant Range (km)** | 1595 | 1890 | 2221 | 2503 | 2808 | 3077 | 3274 |

As the orbital altitude is increased, the number of daylight passes made over the 30 day period also increases. The optimal inclination at which the maximum number of passes is made also increases as orbital altitude is increased. The optimal inclination in all cases was above the target inclination. In each case the difference in inclination between the optimal inclination and the target inclination was close to the value of the Earth central angle for that satellite. The total amount of coverage time and average pass length increased with increasing orbital altitude. The average and maximum slant ranges also increased with increasing altitude.

### *4.2.5 350 km Altitude Circular Orbit, Target Latitude 0 Degrees*

The case of a target site on the equator was tested over a range of inclinations. Each test was run for a 30 day time span and the total number of daylight passes recorded. For each inclination tested the longitude of the ascending node was optimized to provide the highest number of daylight passes. The minimum elevation angle used from the target site to the satellite was zero degrees. Figure 4.5 shows the results for a 350 km altitude circular orbit and a target latitude of zero degrees.

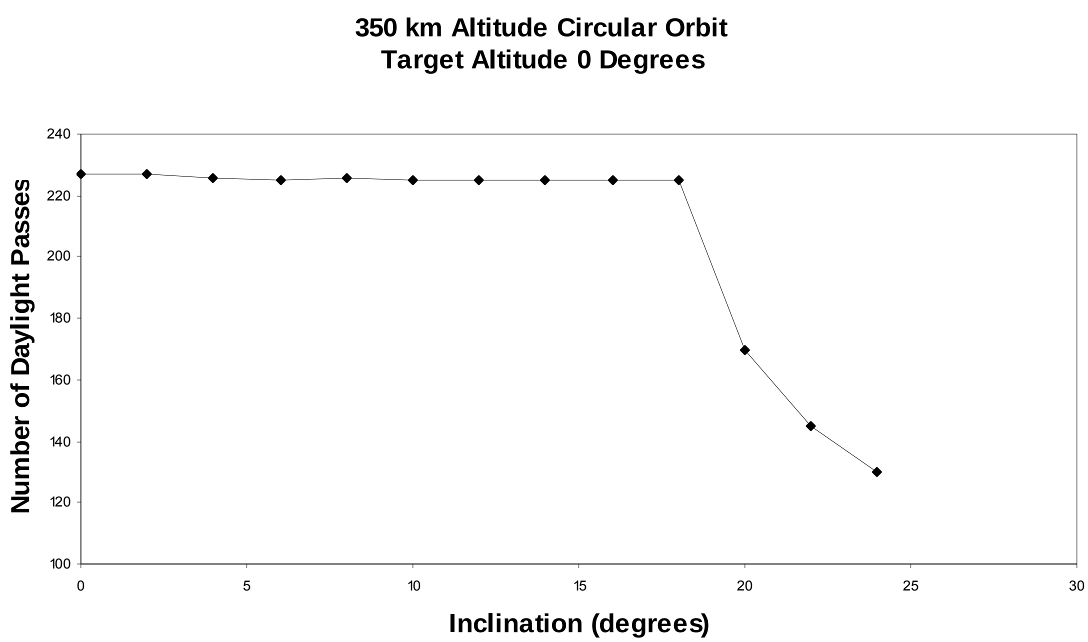


**Fig 4.5 Number of daylight passes made during a 30 day time span vs. inclination 350 km altitude circular orbit, target latitude 0°**

As the inclination is increased above the target latitude there is no significant change in the number of passes until the inclination reaches 18 degrees, at which point

the number of passes drops off steeply as the inclination is increased. The trend suggests there is no improvement in increasing inclination above the target inclination. At an inclination of zero degrees, the satellite makes the maximum number of daylight passes which is 227 passes. For a satellite in an equatorial 350 km altitude orbit, the latitude of the target is always within view of the satellite so no additional benefit can be gained by increasing the inclination. The Earth-central angle is 18.56 degrees for a 350 km altitude circular orbit. As long as the inclination remains at or below 18 degrees, the latitude of the target site should always be within view of the satellite. Figure 4.6 shows the average slant range for each of the inclinations tested.

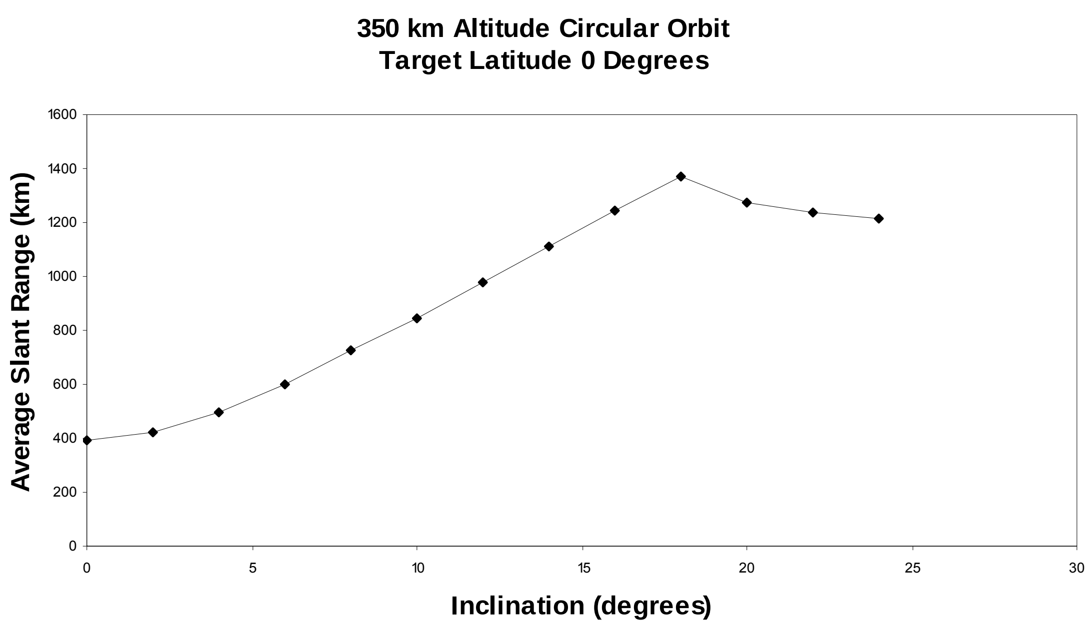


**Fig 4.6 Average slant range for 350 km altitude circular orbit and target latitude 0°**

As the inclination is increased the average slant range also increases until it reaches its maximum at 18 degrees. Although there is little change in the number of passes as long

as the inclination remains below 18 degrees, an inclination of zero offers the additional benefit of having the smallest average slant range to the target site. Although there may not be a significant decrease in the number of daylight passes at inclinations greater than zero, there is a significant increase in the average slant range from the satellite to the target.

#### *4.2.6 800 km Altitude Circular Orbit, Target Latitude 0 Degrees*

An 800 km altitude circular orbit case was also tested with the target site placed on the equator. A range of inclination values was tested and the number of daylight passes measured. Each inclination was tested over a 30 day time span and a minimum elevation angle of zero degrees was used. For each inclination tested the longitude of the ascending node was optimized to provide the highest number of daylight passes. The results for the number of daylight passes are shown in figure 4.7.

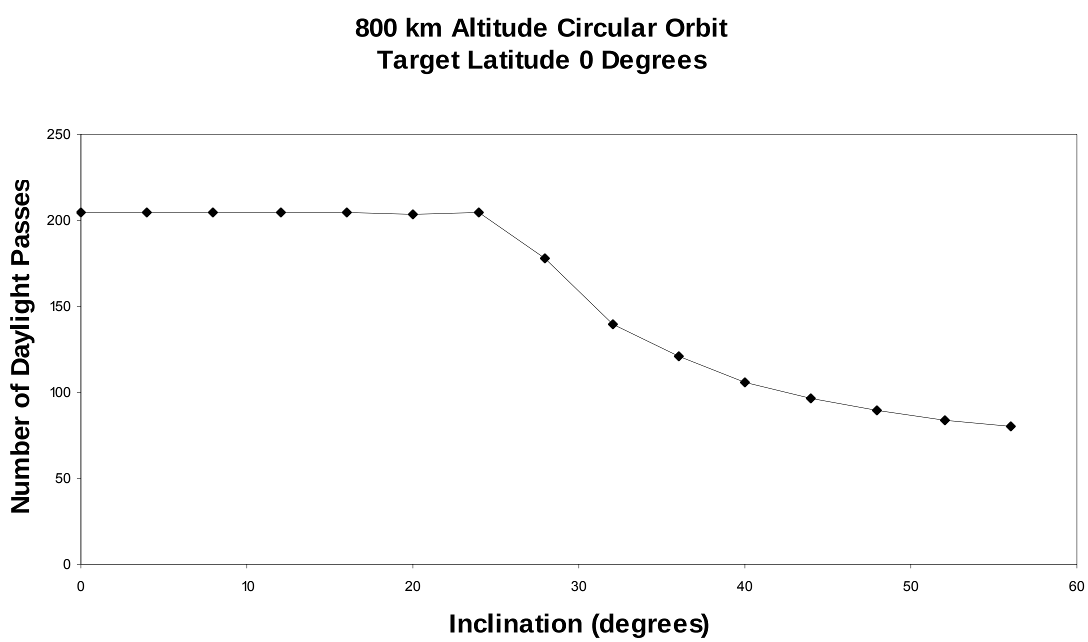


**Fig 4.7 Number of daylight passes made during a 30 day time span vs. inclination 800 km altitude circular orbit, target latitude 0°**

The maximum number of daylight passes made over the 30 days is 205 passes. There is no significant variation in the number of passes made as the inclination is increased from zero to 24 degrees. As the inclination is increased above 24 degrees the number of daylight passes steadily decreases. The trend shown is consistent with the 350 km orbit case. As long as the satellite's inclination is at or below 24 degrees, the target's latitude band will always be within view and there will be little variation in the number of passes. Above an inclination of 24 degrees the number of passes will decrease as the inclination is increased.

Figure 4.8 shows the average slant range versus inclination for this case. The trend is again consistent with the 350 km orbit case. As the inclination is increased, the average slant range increases until it reaches a maximum at 28 degrees. The average slant range is minimized at an inclination of zero degrees.

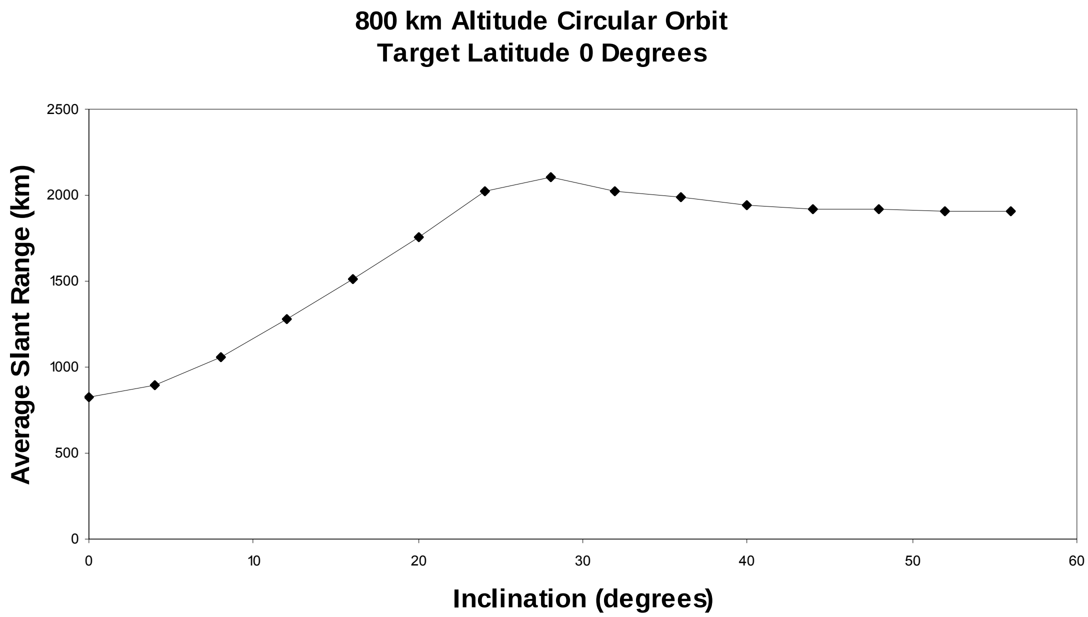


**Fig 4.8 Average slant range for 800 km altitude circular orbit and target latitude 0°**

In comparison with the 350 km altitude orbit case, a fewer number of daylight passes are made by the satellite in an 800 km altitude orbit. Table 4.4 summarizes the coverage properties for a satellite in an equatorial orbit at 350 km altitude and at 800 km altitude.

**Table 4.4 Comparison of Coverage Properties**
**Equatorial Orbits at 350 km and 800 km, Target latitude 0 degrees**

| Orbit Altitude (km) | 350 | 800 |
|---|---|---|
| Number of Daylight Passes | 227 | 205 |
| Total Coverage Time (hours) | 36.6 | 54.4 |
| Average Pass Length (minutes) | 9.7 | 15.9 |
| Average Slant Range to Target (km) | 396 | 830 |
| Maximum Slant Range to Target (km) | 2073 | 2945 |

The 800 km altitude orbit provides fewer total passes but does provide a larger total coverage time due to longer duration passes. The 350 km altitude orbit provides a greater number of passes and a smaller slant range to the target site.

#### *4.2.7 350 km Altitude Circular Orbit, Target Latitude 10 Degrees*

Another case tested was a circular orbit at 350 km altitude and a target site at 10 degrees latitude. A range of inclinations was tested to determine the inclination that provides the maximum number of daylight passes. The tests were run using a 30 day time period and for each inclination tested the longitude of the ascending node was optimized to provide the highest number of daylight passes. The minimum elevation angle used was zero degrees. Figure 4.9 shows the results for the number of daylight passes made during the 30 day period.

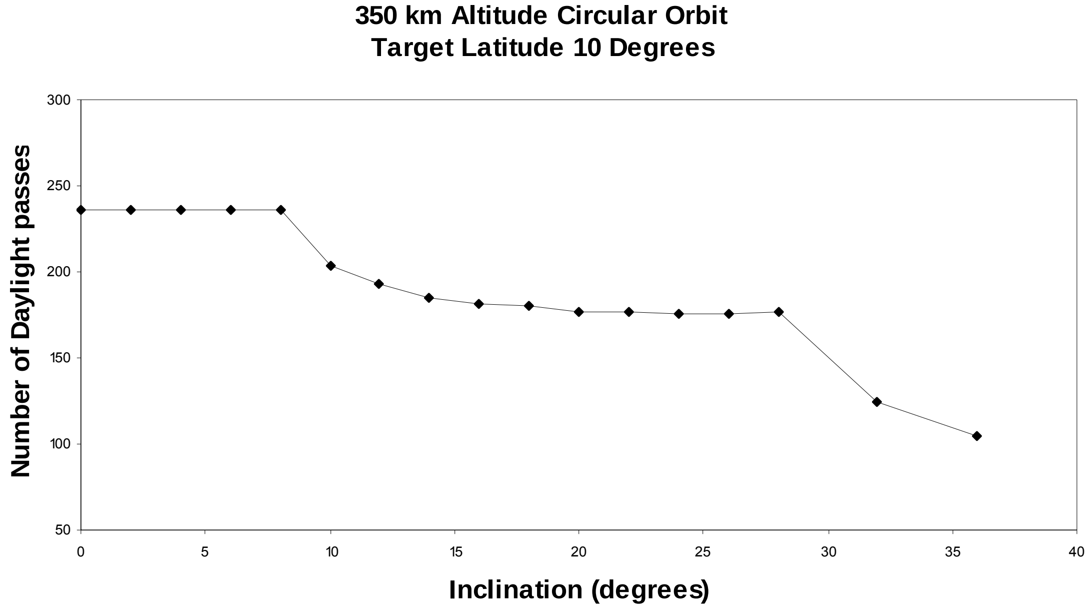


**Fig 4.9 Number of daylight passes made during a 30 day time span vs. inclination 350 km altitude circular orbit, target latitude 10°**

The maximum number of daylight passes made over the 30 day period is 236 passes. At an inclination of zero degrees the number of daylight passes made is the maximum amount. As the inclination increases above 8 degrees, the number of daylight passes begins to steadily decrease. The average slant range is shown in figure 4.10. The slant range is minimized at an inclination of fourteen degrees.

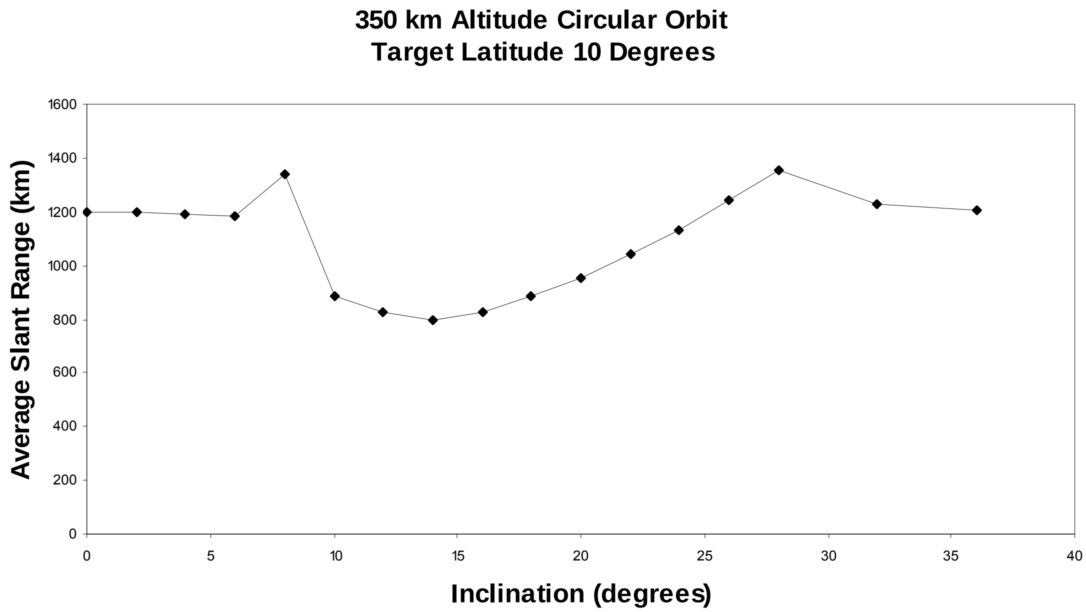


**Fig 4.10 Average slant range for 350 km altitude circular orbit Target latitude 10°**

### *4.2.8 800 km Altitude Circular Orbit, Target Latitude 10 Degrees*

An 800 km altitude orbit was tested with the target placed at a latitude of ten degrees. The tests were run for a time period of 30 days and a minimum elevation angle of zero degrees was used. The longitude of the ascending node for each orbit was optimized to provide the maximum number of daylight passes. For each inclination tested the number of daylight passes made over the target was measured.

Figure 4.11 shows the results for the range of inclinations tested. The maximum number of daylight passes made over the 30 day period is 214 passes. At an inclination of zero the maximum number of passes is made. The number of passes made remains the same until the inclination is increased above 16 degrees, at which point the number of passes decreases and continues to decrease as the inclination is increased.

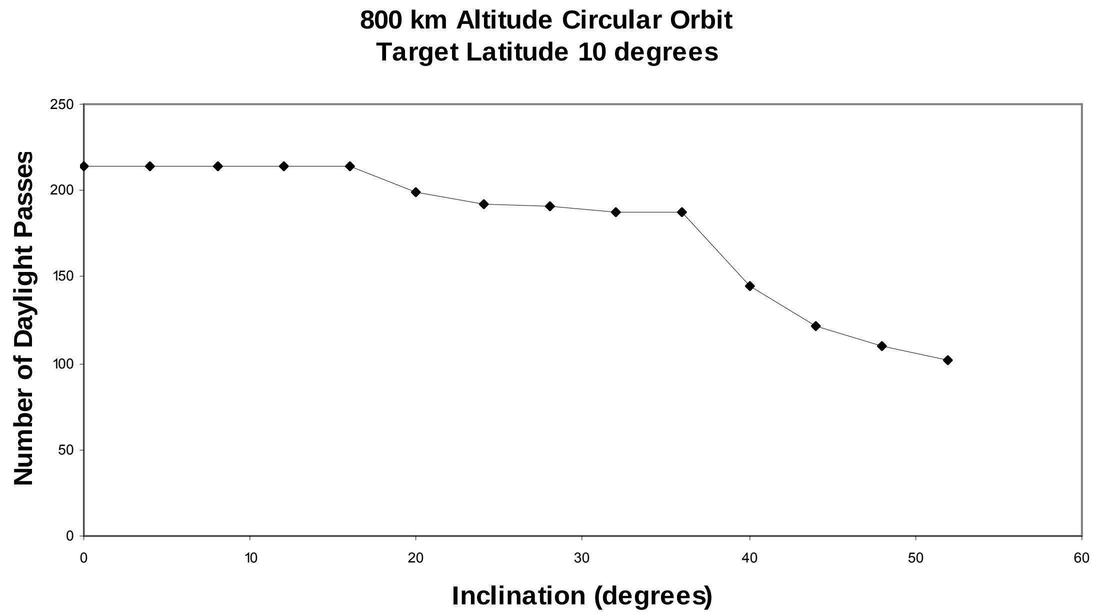


**Fig 4.11 Number of daylight passes made during a 30 day time span vs. inclination 800 km altitude circular orbit, target latitude 10°**

Figure 4.12 shows the average slant range for each inclination tested. The slant range is minimized at an inclination of 12 degrees.

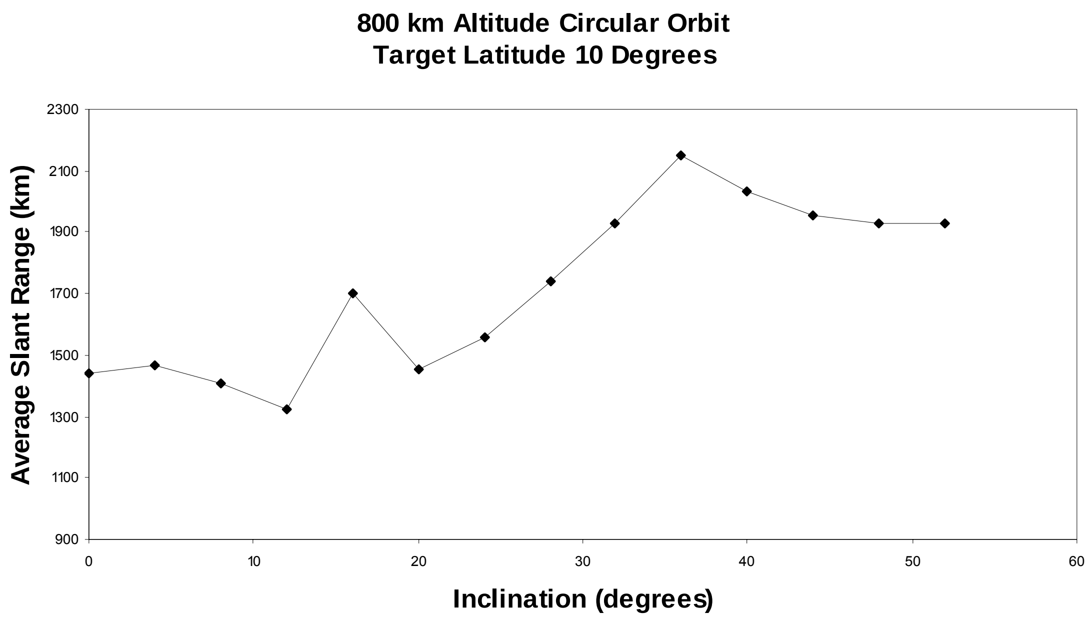


**Fig 4.12 Average Slant Range for 800 km altitude circular orbit Target latitude 10°**

### 4.3 Optimum Inclination for Maximum number of Daylight Passes with Constrained Slant Range

The slant range, measured as the distance from a satellite to a target along its line of sight, may be an important factor for satellite operations. The surveillance tools onboard the satellite may have a maximum distance at which they can effectively operate. When this is the case, any passes made over the target will only be useful if the slant range to the target is less than maximum distance required by the equipment. In order to examine the impact of a maximum slant range on inclination optimization, several cases were run with a constraint placed on the slant range. Only satellite passes made with a slant range less than the constraint were counted during the simulations.

### *4.3.1 350 km Altitude Circular Orbit, Target at Latitude 33 Degrees, Maximum Slant Range 800 km*

The first case tested was a 350 km altitude circular orbit and a target at a latitude of 33 degrees. The slant range constraint chosen was a maximum slant range of 800 km. A range of inclinations was tested to see where the maximum number of daylight passes occurs. For each inclination tested the longitude of ascending node was optimized to provide the maximum number of daylight passes. Each test was run for a time period of thirty days.

Figure 4.13 shows the number of daylight passes made over the thirty day period.

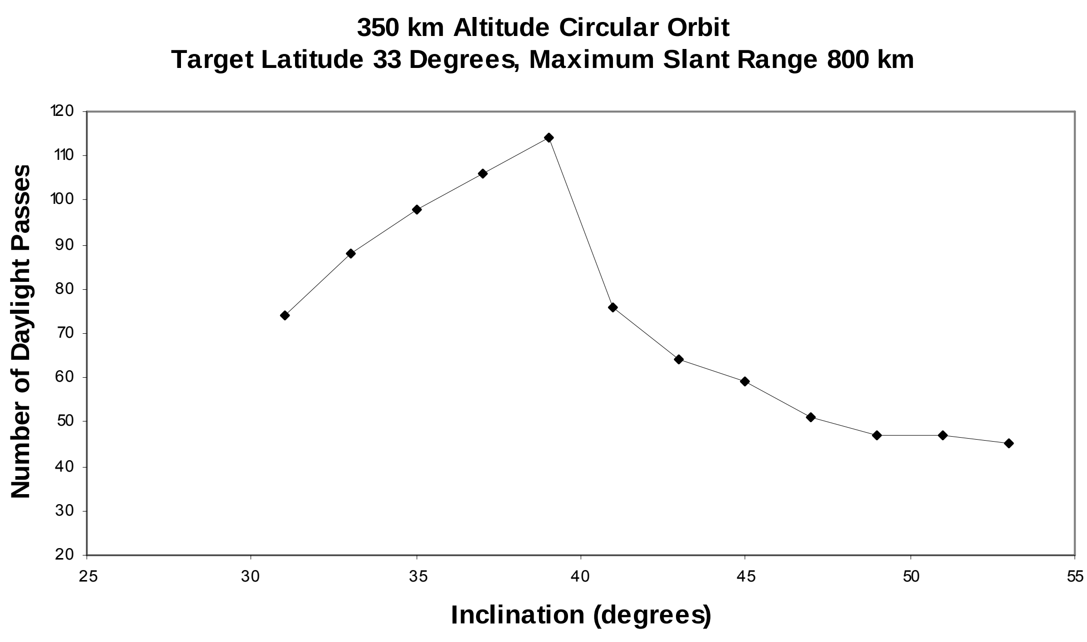


**Fig 4.13 Number of daylight passes made during a 30 day time span vs. inclination 350 km altitude circular orbit, target latitude 33°, slant range maximum 800 km**

As the inclination is increased above the latitude of the target, the number of daylight passes increases until it reaches a maximum at 39 degrees. The maximum number of

passes made over the 30 days is 114 passes at an inclination of 39 degrees. As the inclination is increased above 39 degrees, the number of passes made decreases. Table 4.5 summarizes the coverage properties of a satellite in an orbit at an inclination of 39 degrees.

**Table 4.5 Summary of Coverage Properties**
**350 km Altitude Circular Orbit, 39° Inclination, Target Latitude 33°**
**Maximum Slant Range Constraint 800 km**

| | |
|---|---|
| Number of Daylight Passes | 114 |
| Total Coverage Time (hours) | 18.3 |
| Average Pass Length (minutes) | 9.6 |
| Average Slant Range to Target (km) | 594 |
| Maximum Slant Range to Target (km) | 800 |

The average slant range is 594 km which is well below the constraint of 800 km. In comparison with the unconstrained case, the number of passes made is less. At an inclination of 39 degrees, the unconstrained case of a 350 km altitude orbit would have yielded 158 passes. With the slant range constraint, 114 passes are made in which the slant range requirement is met and 44 passes are made at a distance that exceeds 800 km. The maximum number of passes occurs at an inclination of 39 degrees in comparison with 51 degrees for the unconstrained case. For a satellite at an altitude of 350 km and using a maximum slant range of 800 km, the effective Earth central angle is approximately 6.3 degrees as depicted in figure 4.14 and in this case the optimum inclination occurs at 6 degrees above the target latitude; again the results are consistent with the analytical analysis.

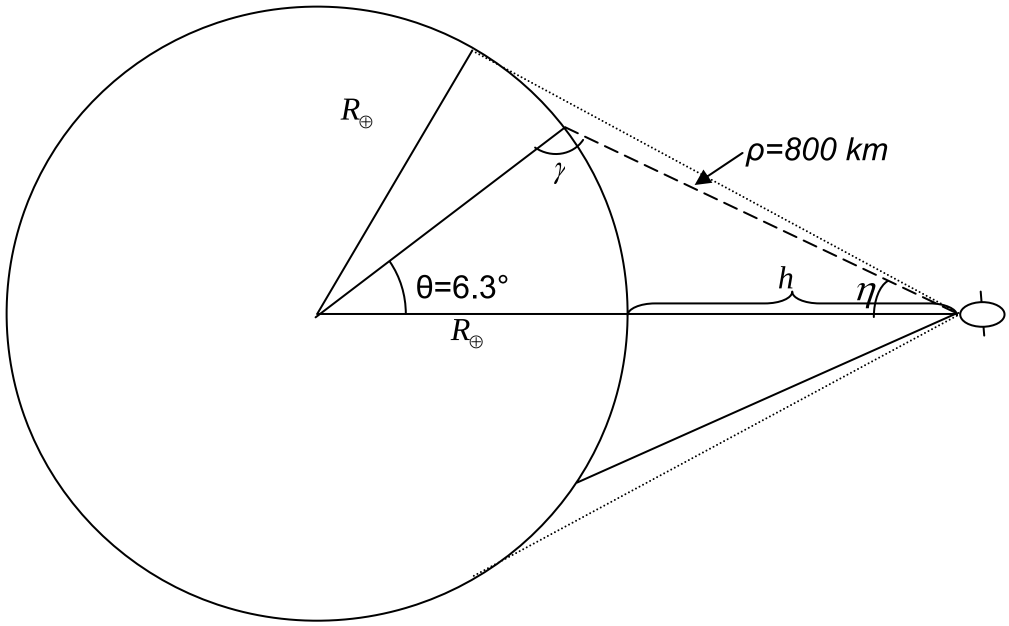


**Fig 4.14 Effective Earth Central Angle for Constrained Slant Range**

### *4.3.2 Varying Altitudes with Slant Range Constraint of 800 km*

Table 4.6 compares the coverage properties of various altitudes with a maximum slant range constraint of 800 km. A target latitude of 33 degrees and a minimum elevation angle of zero was used and a time period of 30 days. For each altitude, the inclination has been optimized to yield the maximum number of passes.

**Table 4.6 Comparison of Coverage Properties for Varying Altitudes Maximum Slant Range Constraint 800 km, Target Latitude 33°**

| Orbital Altitude (km) | 200 | 300 | 400 | 500 | 600 | 700 |
|---|---|---|---|---|---|---|
| Optimal Inclination (degrees) | 39 | 39 | 38 | 38 | 37 | 36 |
| Effective Earth Central Angle +Target Latitude | 39.9 | 39.5 | 39 | 38.4 | 37.5 | 36.3 |
| Number of Daylight Passes | 121 | 118 | 108 | 108 | 96 | 79 |
| Total Coverage Time (hours) | 14 | 17.3 | 19.1 | 21.6 | 21.6 | 19.5 |
| Average Pass Length (minutes) | 7 | 8.8 | 10.6 | 12 | 13.5 | 14.8 |
| Average Slant Range (km) | 527 | 569 | 579 | 652 | 697 | 752 |
| Maximum Slant Range (km) | 796 | 797 | 786 | 789 | 798 | 799 |

As the altitude increases, the number of passes made decreases. The optimum altitude and inclination for the maximum number of passes was 200 km and 39 degrees. At higher altitudes, the number of passes is decreased due to the slant range constraint. However at higher altitudes the total coverage time over the target increases as the average pass length increases. If total coverage time was selected as the figure of merit a higher altitude would have been selected as the optimum altitude. Depending on the metric or metrics chosen to evaluate coverage, the optimum orbit parameters will vary.

### 4.4 Optimum Inclination for Maximum Number of Daylight Passes with Constrained Elevation Angle

The elevation angle, measured from the local horizon of the target to the line of sight vector to the satellite, may have a minimum value that is greater than zero. All satellites are limited by the horizon but often a higher elevation angle is also required. For visible imaging satellites, an elevation angle of ninety degrees is ideal because the image will be taken directly overhead. As the elevation angle decreases, the images will be more difficult to interpret and less useful. At very small elevation angles, objects may obstruct the line of sight of the satellite to the target and prevent it from operating. In order to examine the impact of a minimum elevation requirement, several cases were run with a constraint on the minimum elevation angle required to view the target. If the elevation angle was smaller than the constraint value, the target was not considered visible to the satellite.

#### *4.4.1 350 km Altitude Circular Orbit, Target at Latitude 33 Degrees, Minimum Elevation Angle 10 Degrees*

The first case examined was a 350 km altitude circular orbit with the target placed at a latitude of 33 degrees. A minimum elevation angle of 10 degrees was used. In order to determine the inclination at which the maximum number of daylight passes occurs, a range of inclinations was tested. A time period of 30 days was used and the longitude of the ascending node was optimized to provide the maximum number of daylight passes for each inclination tested.

Figure 4.15 shows the number of daylight passes made at each inclination tested. The maximum number of daylight passes made over the 30 day period is 144 and occurs at an inclination of 43 degrees.

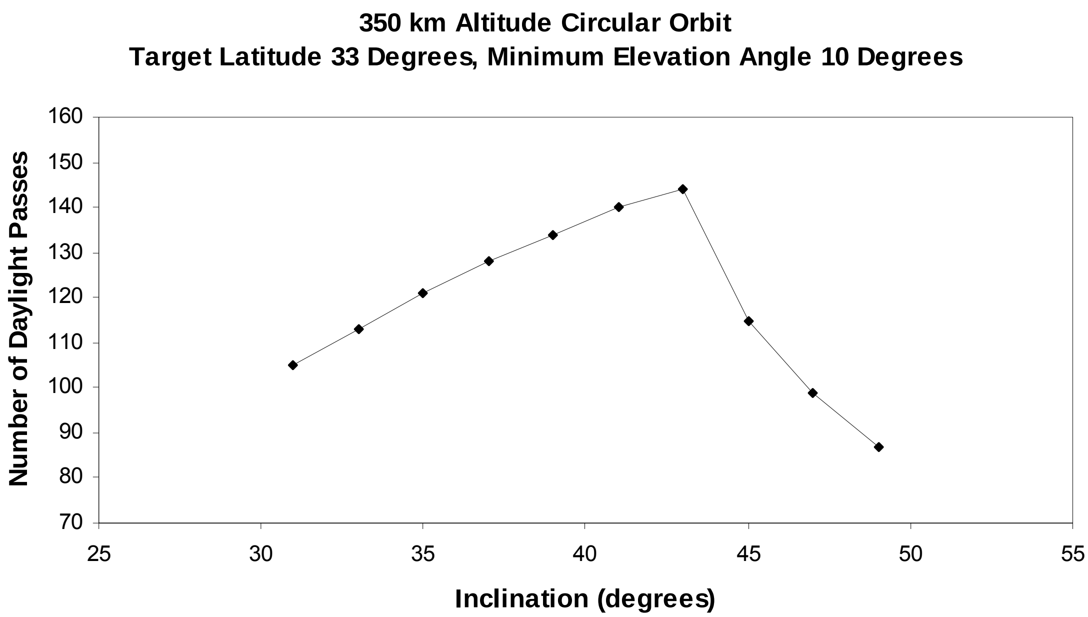


**Fig 4.15 Number of daylight passes made during a 30 day time span vs. inclination 350 km altitude circular orbit, target latitude 33°, minimum elevation angle 10°**

As the inclination is increased above the target latitude, the number of passes increases until it reaches a maximum at an inclination of 43 degrees. As the inclination increases above 43 degrees, the number of daylight passes decreases.

Figure 4.16 shows the average slant range for the range of inclinations tested. The maximum average slant range occurs at an inclination of 43 degrees.

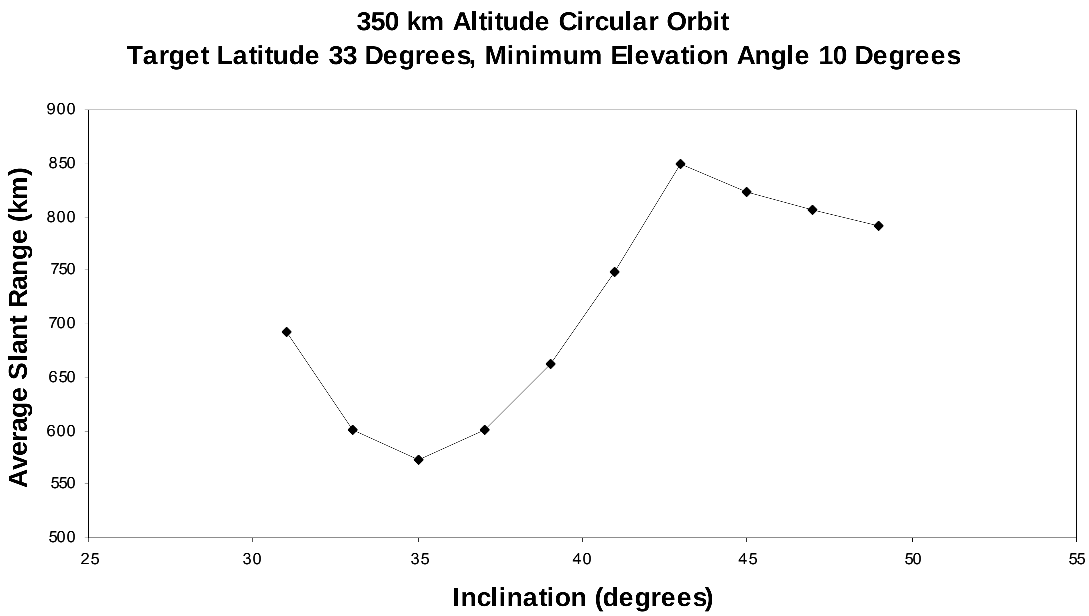


**Fig 4.16 Average Slant Range for 350 km altitude circular orbit, target latitude 33° Minimum elevation angle 10°**

The inclination which provides the highest number of daylight passes also has the highest average slant range.

Table 4.7 provides additional information about the coverage of a satellite at an inclination of 43 degrees. In comparison with the case where a minimum elevation of zero was used, the number of passes made is less. This is an expected result since a higher elevation angle creates a smaller field of view for the satellite. The optimum inclination occurs at a value of 43 degrees as opposed to 51 degrees for the unconstrained case.

**Table 4.7 Summary of Coverage Properties**
**350 km Altitude Circular Orbit, 43° Inclination, Target Latitude 33°**
**Minimum Elevation Angle Constraint 10°**

| | |
|---|---|
| Number of Daylight Passes | 144 |
| Total Coverage Time (hours) | 10.3 |
| Average Pass Length (minutes) | 4.3 |
| Average Slant Range to Target (km) | 849 |
| Maximum Slant Range to Target (km) | 1296 |

For a satellite in a circular orbit at an altitude of 350 km and with a minimum elevation angle requirement of ten degrees, the effective Earth central angle is approximately 11 degrees as depicted in figure 4.17. For this case the optimum inclination occurs at 10 degrees above the target latitude.

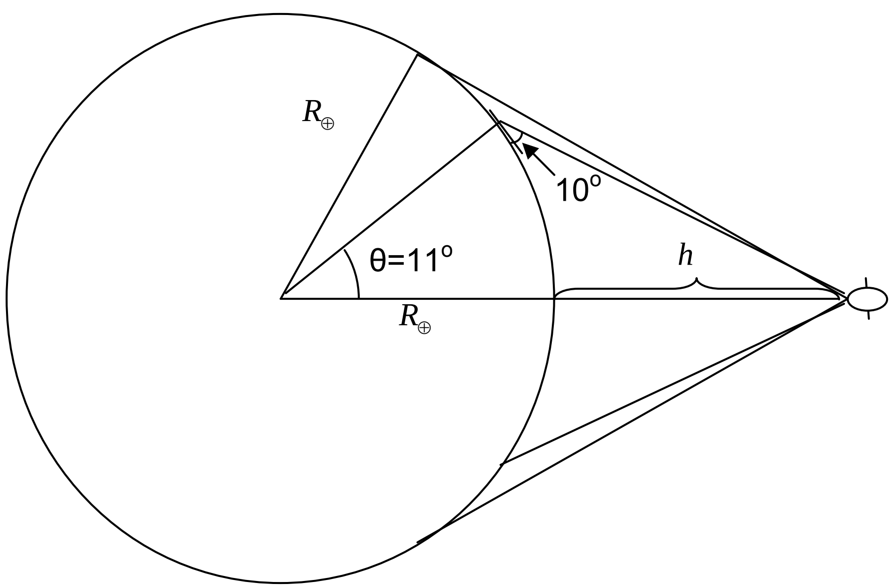


**Fig 4.17 Effective Earth central angle for constrained elevation angle**

### 4.4.2 800 km Altitude Circular Orbit, Target at Latitude 33 Degrees, Minimum Elevation Angle 10 Degrees

An 800 km altitude orbit was also tested with the target latitude at 33 degrees and a minimum elevation angle of 10 degrees. A range of inclinations was tested and the number of daylight passes measured. Each inclination was tested over a 30 day period. The longitude of the ascending node was optimized for each inclination to provide the highest number of daylight passes. Figure 4.18 shows the results for the range of inclination values tested.

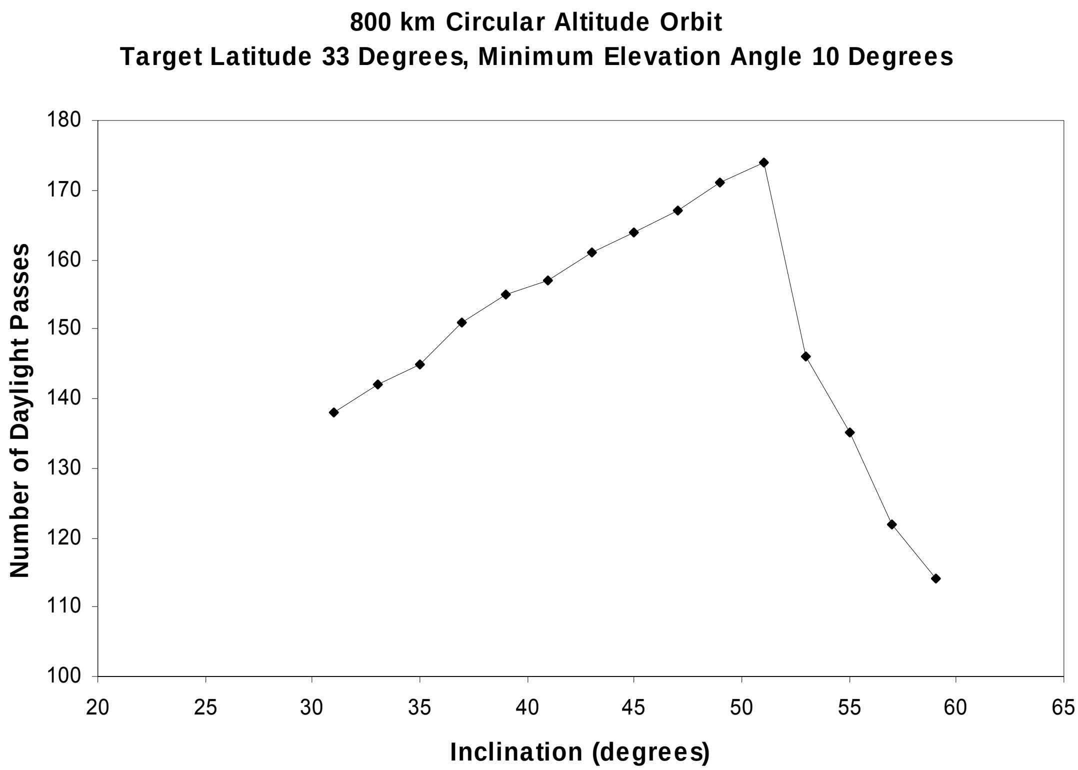


**Fig 4.18 Number of daylight passes made during a 30 day time span vs. inclination 800 km altitude circular orbit, target latitude 33°, minimum elevation angle 10°**

The maximum number of daylight passes made over the thirty day period is 174 passes at an inclination of 51 degrees. The number of daylight passes increases as the inclination increases above the target latitude until it reaches a maximum at an inclination of 51 degrees. The number of passes decreases as the inclination is increased above 51 degrees. The effective Earth central angle for an orbit altitude of 800 km and a minimum Earth central angle of 10 degrees is approximately 18.9 degrees. For this case the optimum inclination was 18 degrees above the target latitude.

Figure 4.19 shows the average slant range for the range of inclinations tested. The maximum average slant range occurs at an inclination of 51 degrees. The inclination which provides the maximum number of daylight passes is also the inclination with the highest average slant range.

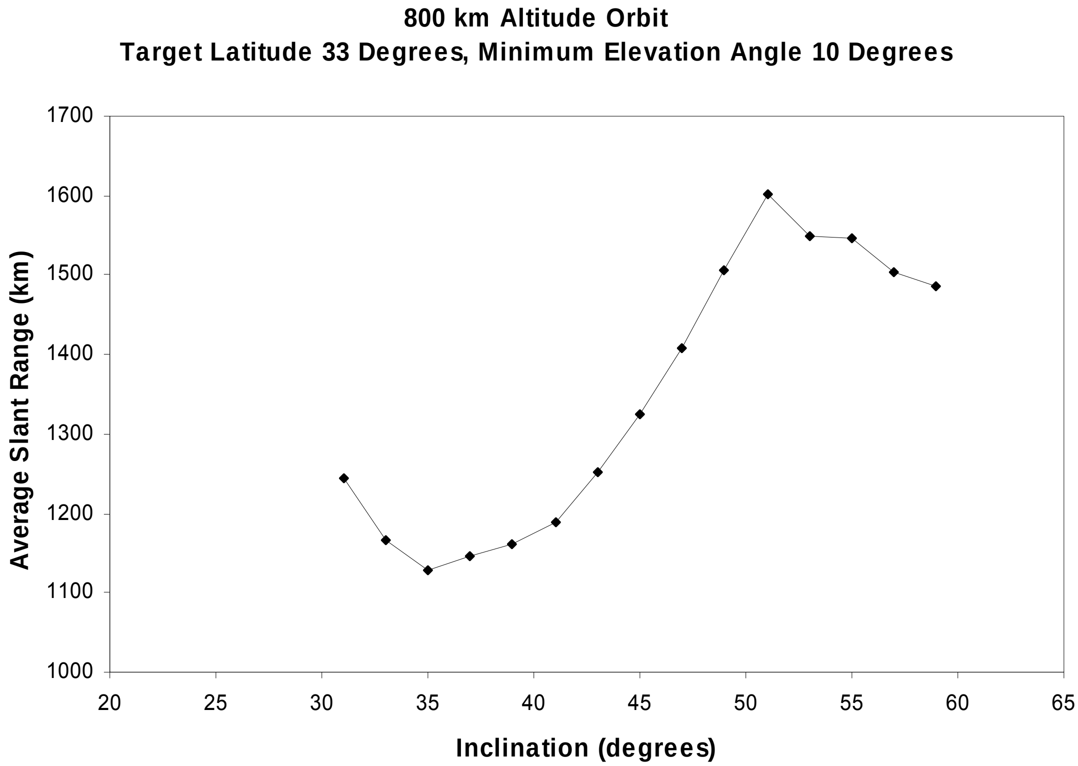


**Fig 4.19 Average Slant Range for 800 km altitude circular orbit, target latitude 33° Minimum elevation angle 10°**

Table 4.8 provides a summary of the coverage properties at an inclination of 51 degrees. In comparison with a minimum elevation angle of zero, the total number of passes made is less which is expected since a larger elevation angle decreases the field of view of the satellite. In comparison with the 350 km case with a minimum elevation angle of 10 degrees, the total number of passes made is higher which also expected since increasing orbital altitude has been shown to yield a higher number of passes.

**Table 4.8 Summary of Coverage Properties**
**800 km Altitude Circular Orbit, 51° Inclination, Target Latitude 33°**
**Minimum Elevation Angle Constraint 10°**

| | |
|---|---|
| Number of Daylight Passes | 174 |
| Total Coverage Time (hours) | 23.1 |
| Average Pass Length (minutes) | 8 |
| Average Slant Range to Target (km) | 1601 |
| Maximum Slant Range to Target (km) | 2364 |

## 4.5 Elliptical Orbits

In order to compare the performance of an elliptical orbit with a circular orbit, a test case was examined using an elliptical orbit. An elliptical orbit with a semi-major axis of 6,728 km was tested. This semi-major axis value is the same as the semi-major axis of a circular orbit with an orbit altitude of 350 km. The elliptical orbit tested had a perigee altitude of 300 km and an apogee altitude of 400 km. The target was placed at a latitude of 33 degrees and a minimum elevation angle of zero degrees was used. A range of inclinations was tested and the number of daylight passes measured over a thirty day time period. The longitude of the ascending node and argument of perigee were optimized to provide the highest number of daylight passes.

The maximum number of daylight passes made over the thirty day time period was 188 passes at an inclination of 51 degrees. Table 4.9 summarizes the coverage parameters for the elliptical orbit at an inclination of 51 degrees and compares them with the coverage properties of a circular orbit at an altitude of 350 km.

**Table 4.9 Summary of Coverage Properties**
**300x400 km Elliptical Orbit vs. 350 km Altitude Circular Orbit, 51° Inclination**
**Target Latitude 33°**

| Orbit | Elliptical | Circular |
|---|---|---|
| Number of Daylight Passes | 188 | 186 |
| Total Coverage Time (hours) | 23.3 | 21.1 |
| Average Pass Length (minutes) | 7.4 | 6.8 |
| Average Slant Range to Target (km) | 1404 | 1381 |
| Maximum Slant Range to Target (km) | 2270 | 2102 |

The elliptical orbit had a slightly higher number of daylight passes made but not significantly higher. The average and maximum slant ranges were higher for the elliptical orbit. The total coverage time and average pass length were higher for the elliptical orbit.

## 4.6 Orbit Optimization

***Scenario 1***

An orbit altitude range of 200 km to 800 km was used as search space for the optimization algorithm. A minimum elevation angle of ten degrees and a simulation period of 30 days were used. The target site was located at a latitude of 33 degrees. Table 4.10 shows the solution space bounds for the scenario.

**Table 4.10 Upper and Lower Bounds for Solution Space**
**Minimum Elevation Angle 10°, Target Latitude 33°**

| | **Lower Bound** | **Upper Bound** |
|---|---|---|
| **Orbital Altitude (km)** | 200 | 800 |
| **Orbital Inclination (degrees)** | 34 | 51 |
| **Number of Daylight Passes** | 97 | 171 |
| **Average Slant Range (km)** | 369 | 1597 |

Table 4.11 shows the results for several optimization cases with various weights on the number of passes and average slant range. The selected orbit parameters for each case are shown as well as the coverage properties for each selected orbit.

**Table 4.11 Optimization Results for Minimum Elevation Angle 10 degrees**

| **Optimization Parameters** | | | | | | | |
|---|---|---|---|---|---|---|---|
| Pass Weight | 5 | 4 | 8 | 6 | 8 | 5 | 6 |
| Range Weight | 5 | 8 | 4 | 8 | 6 | 6 | 5 |
| Weighting Ratio (pass weight/range weight) | 1 | 0.5 | 2 | .75 | 1.33 | .83 | 1.2 |
| **Optimized Orbit Parameters** | | | | | | | |
| Altitude (km) | 500 | 200 | 750 | 200 | 500 | 350 | 500 |
| Inclination (degrees) | 45 | 37 | 51 | 39 | 45 | 39 | 45 |
| Longitude of Ascending Node (degrees) | 72 | 108 | 72 | 108 | 72 | 108 | 72 |
| **Orbit Coverage Properties** | | | | | | | |
| Number of Daylight Passes | 159 | 111 | 175 | 123 | 159 | 132 | 159 |
| Average Slant Range (km) | 1068 | 433 | 1577 | 535 | 1068 | 655 | 1068 |
| Total Coverage Time (hours) | 16.1 | 6.1 | 20.6 | 5.9 | 16.1 | 11.3 | 16.1 |
| Average Pass Length (minutes) | 6.1 | 3.3 | 7 | 2.9 | 6.1 | 5.1 | 6.1 |
| Maximum Slant Range (km) | 1494 | 820 | 2262 | 856 | 1494 | 1265 | 1494 |

***Scenario 2***

An orbit altitude range of 200 km to 800 km was used as search space for the optimization algorithm. A minimum elevation angle of zero degrees and a simulation period of 30 days were used. The target site was located at a latitude of 33 degrees. Table 4.12 shows the solution space bounds for the scenario.

**Table 4.12 Upper and Lower Bounds for Solution Space**
**Minimum Elevation Angle 0°, Target Latitude 33°**

| | **Lower Bound** | **Upper Bound** |
|---|---|---|
| **Orbital Altitude (km)** | 200 | 800 |
| **Orbital Inclination (degrees)** | 35 | 60 |
| **Number of Daylight Passes** | 131 | 203 |
| **Average Slant Range (km)** | 556 | 2140 |

Table 4.13 shows the results for several optimization cases with various weights on the number of passes and average slant range. The selected orbit parameters for each case are shown as well as the coverage properties for each selected orbit.

**Table 4.13 Optimization Results for Minimum Elevation Angle 0 degrees**

| **Optimization Parameters** | | | | | |
|---|---|---|---|---|---|
| Pass Weight | 5 | 3 | 6 | 6 | 8 |
| Range Weight | 5 | 6 | 3 | 8 | 6 |
| Weighting Ratio (pass weight/range weight) | 1 | .5 | 2 | .75 | 1.3 |
| **Optimized Orbit Parameters** | | | | | |
| Altitude (km) | 500 | 200 | 750 | 500 | 500 |
| Inclination (degrees) | 49 | 39 | 59 | 41 | 49 |
| Longitude of Ascending Node (degrees) | 72 | 72 | 36 | 72 | 72 |
| **Orbit Coverage Properties** | | | | | |
| Number of Daylight Passes | 196 | 146 | 203 | 178 | 196 |
| Average Slant Range (km) | 1452 | 675 | 2066 | 1108 | 1452 |
| Total Coverage Time (hours) | 31.4 | 15.5 | 40 | 31.5 | 31.4 |
| Average Pass Length (minutes) | 9.6 | 6.4 | 10.6 | 10.6 | 9.6 |
| Maximum Slant Range (km) | 2407 | 1575 | 3142 | 2580 | 2407 |

### 4.7 Constellation Design

Figure 4.20 shows the distribution of passes made over a 30 day period for a satellite in a 500 km circular orbit with a minimum elevation angle of 10 degrees and a target at a latitude of 33 degrees. On each day the passes are made in succession with approximately 95 minutes, the period of the orbit, between passes. The number of passes made per day ranges from a maximum of 6 to a minimum of 4.

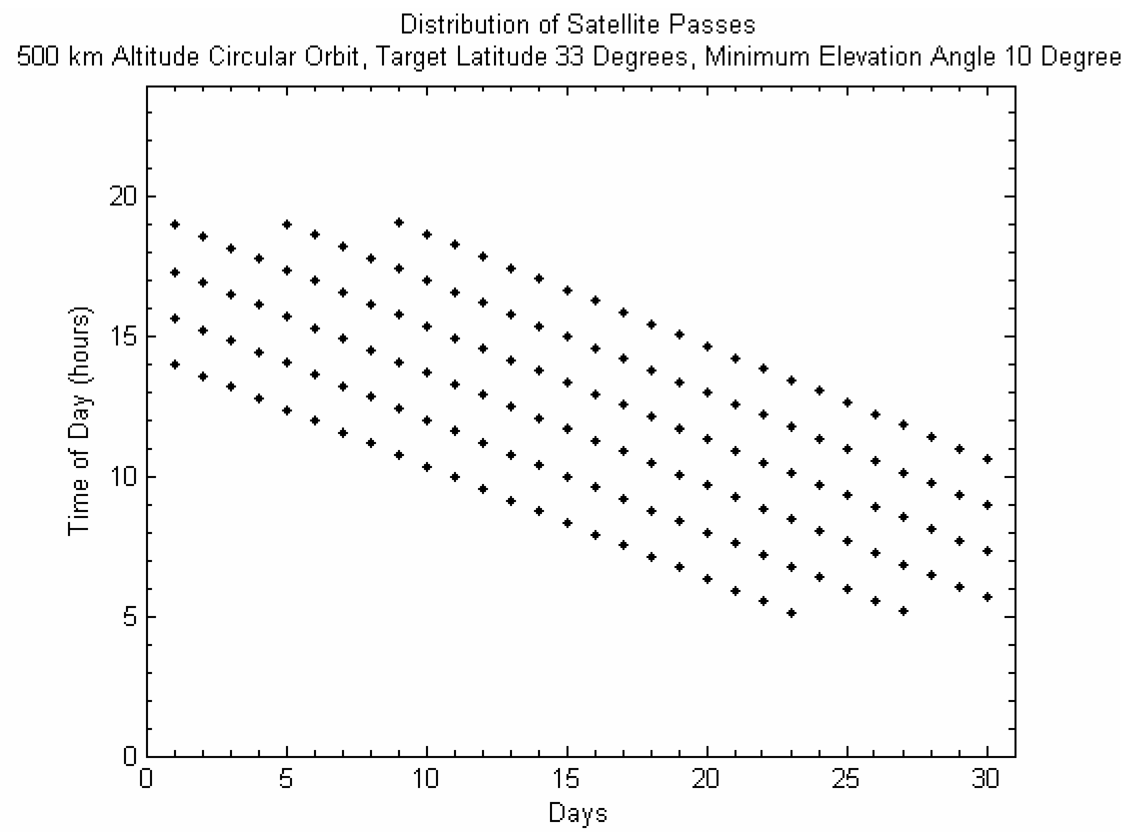


**Figure 4.20 Distribution of Satellite Passes Made Over 30 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Longitude of Ascending Node 72°**
**Target Latitude 33°, Minimum Elevation Angle 10°**

The time of day when the passes occur drifts as the orbit's node regresses. At the beginning of the time period, the passes occur later in the day but on subsequent days the

passes occur at an earlier time of day until at the end of the 30 day period the passes are occurring in the early portion of the day. The impact of this trend is that there are fewer passes on days near the beginning and end of the time period.

#### *4.7.1 Two Satellites Separated by 180 degrees in Mean Anomaly*

One constellation design is to place two satellites in the same orbit but separated by mean anomaly. Figure 4.21 shows the distribution of satellite passes for two satellites separated by 180 degrees of mean anomaly. Both satellites were in 500 km altitude circular orbits at an inclination of 45 degrees and with a longitude of the ascending node of 72 degrees. A minimum elevation angle of 10 degrees was used and the target was located at a latitude of 33 degrees.

The number of passes per day ranges from a maximum of 12 to a minimum of 7. The passes each day occur successively with approximately 47 minutes between each pass. The number of passes has nearly doubled from 159 passes for one satellite to 316 passes with the additional satellite. Similar to the one satellite case, there are fewer passes on days at the beginning and end of the 30 day period.

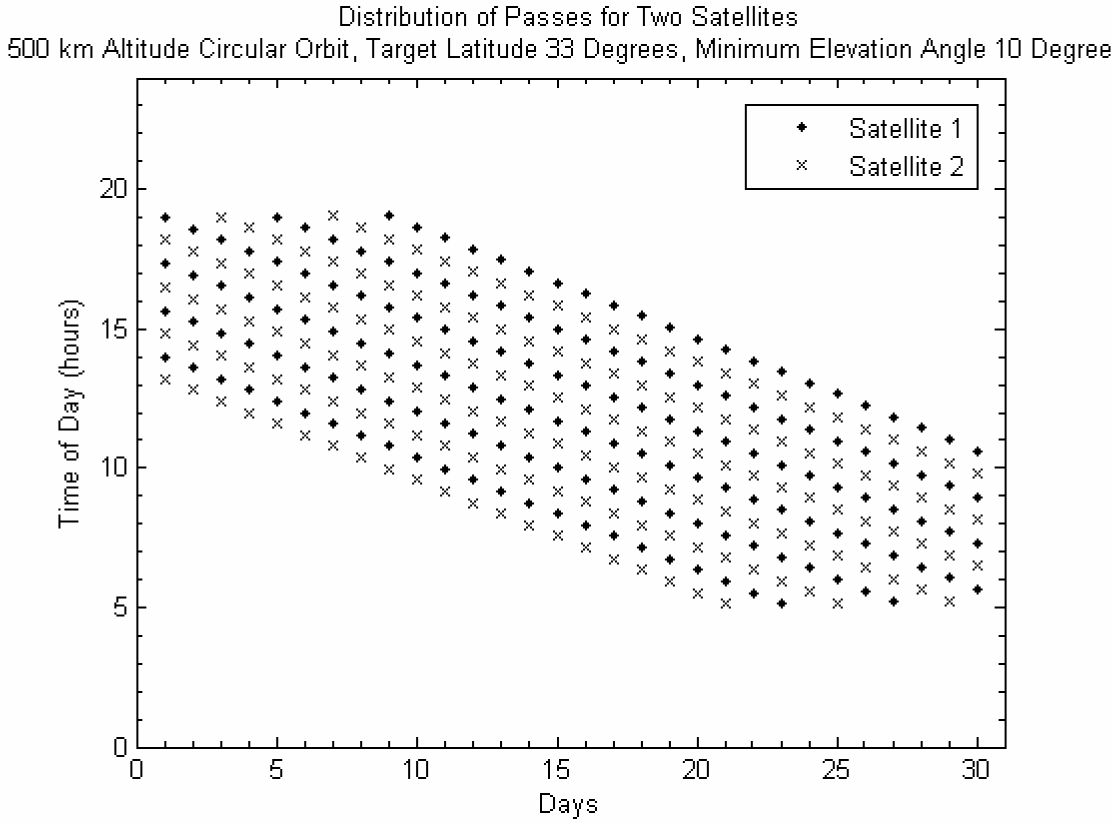


**Figure 4.21 Distribution of Passes for Two Satellites Over 30 Days**
**Satellite 1 and 2: 500 km Altitude Circular Orbit, 45° Inclination, 72° Longitude of Ascending Node, Minimum Elevation Angle 10°, Target Latitude 33°**

### *4.7.2 Two Satellites Separated by 36 Degrees in Longitude of Ascending Node*

Another constellation design method is to place the satellites at the same altitude and inclination but separate the orbits by ascending node. Placing an additional satellite at a different node will spread the times of the satellite passes over a larger span of the day. The larger the spacing between nodes, the more spread out the passes will be. However choosing a node that is too far from the optimized node will mean the second satellite makes very few passes because it will be making passes over the target when the target is not in daylight.

Figure 4.22 shows the number of passes made versus longitude of ascending node for a satellite in a 500 km altitude orbit with an inclination of 45 degrees.

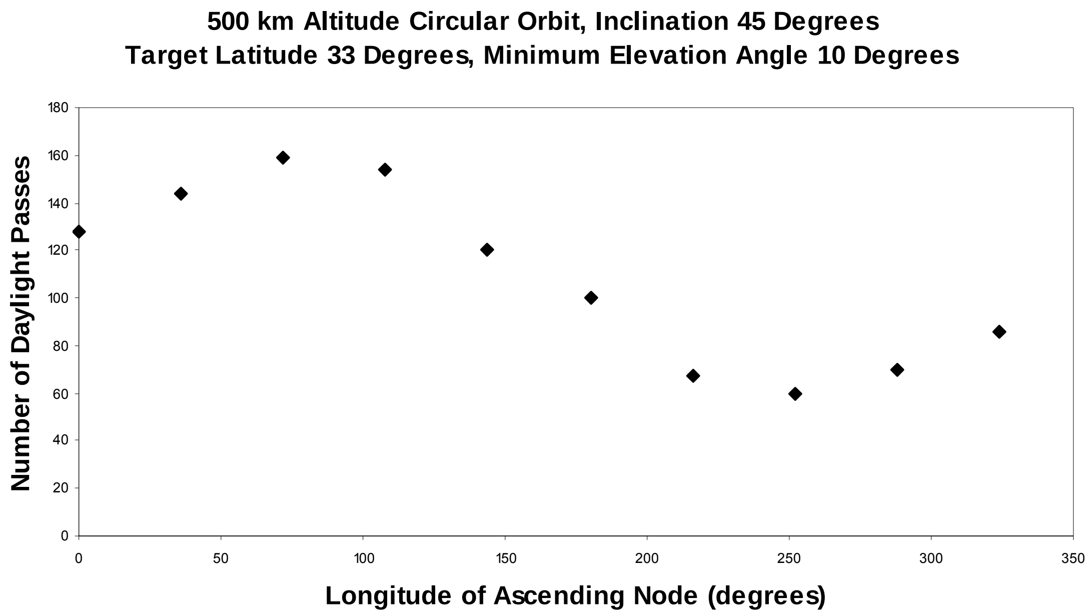


**Figure 4.22 Number of Daylight Passes vs. Longitude of Ascending Node 500 km Altitude Circular Orbit, 45° Inclination, Target Latitude 33° Minimum Elevation Angle 10°**

Adding another satellite at a node of 36 degrees or 108 degrees will provide a high number of additional passes. Figure 4.23 shows the results for a two satellite constellation with one satellite in an orbit with the longitude of ascending node at 72 degrees and the other satellite in an orbit with the longitude of ascending node at 108 degrees. The satellites are also separated by 180 degrees of mean anomaly.

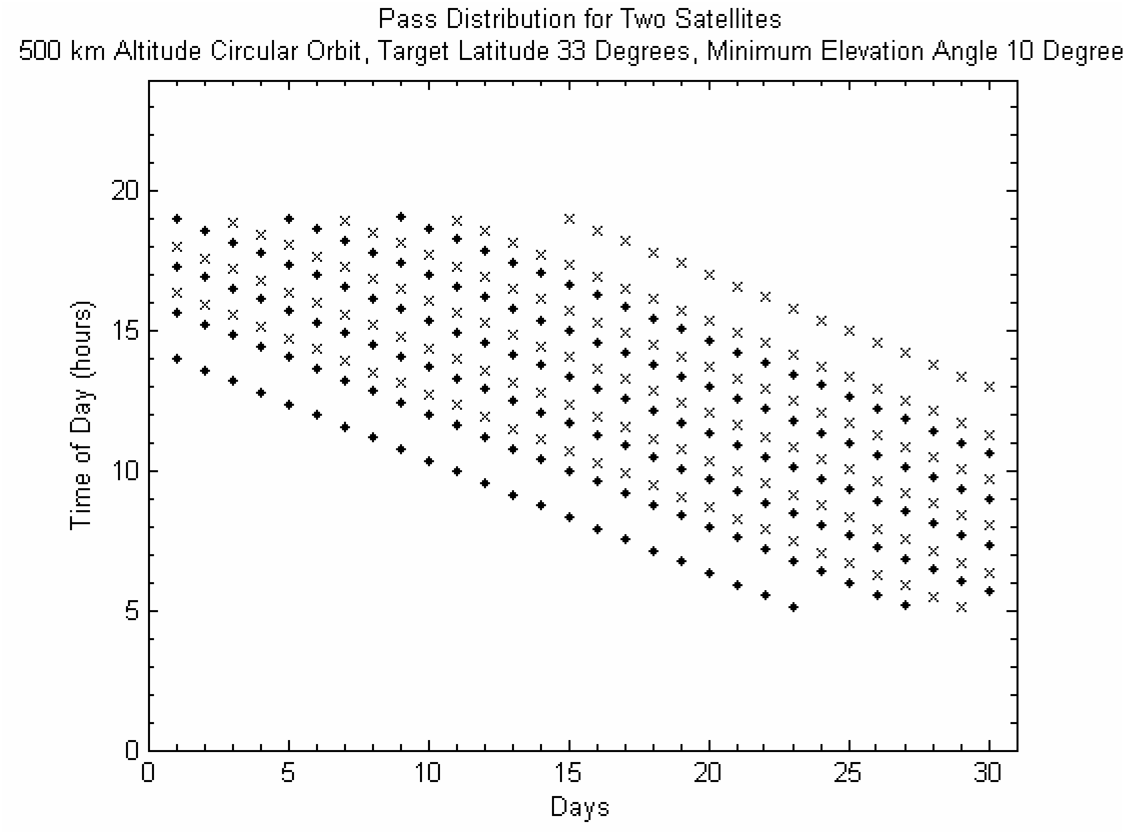


**Figure 4.23 Pass Distribution for Two Satellites Over 30 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Minimum Elevation Angle 10°**
**Satellite 1: 72° Longitude of Ascending Node**
**Satellite 2: 108° Longitude of Ascending Node**

The number of passes made each day ranges from a maximum of 12 to a minimum of 6. The additional satellite's node is shifted to a larger value which means the time of day at which passes are made is shifted as well. The result of this shift is that at the end of the 30 day period, there is less of a decrease in the number of passes made per day. On the final day there are 9 passes made, as compared to 7 passes made on the final day of the two satellites case with both satellites at the same longitude of the ascending node. However there are fewer passes made on the days near the beginning of the 30 day period. Only 6 passes are made on the first day as opposed to 8 passes for the two satellite case with both satellites at the same longitude of the ascending node. The total

number of passes made during the 30 day period is 307 passes in comparison to 316 passes for the case with both satellites at the same longitude of ascending node.

Figure 4.24 shows the results for a two satellite constellation with one satellite in an orbit with the longitude of ascending node at 72 degrees and the other satellite in an orbit with the longitude of ascending node at 36 degrees. The satellites are also spaced by 180 degrees in mean anomaly.

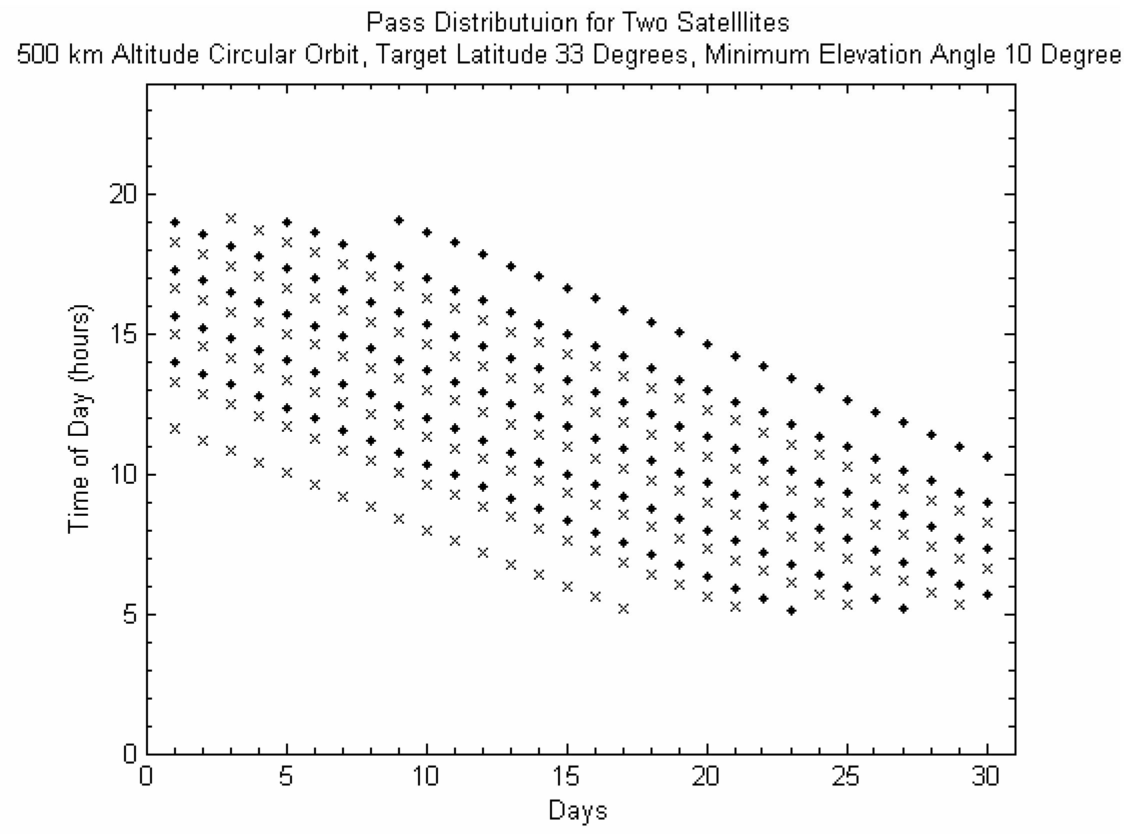


**Figure 4.24 Pass Distribution for Two Satellites Over 30 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Minimum Elevation Angle 10°**
**Satellite 1: 72° Longitude of Ascending Node**
**Satellite 2: 36° Longitude of Ascending Node**

Again the shift in the node value causes a shift in the time of day at which passes occur. The result is there are more passes made at the beginning of the 30 day period than for the case where both satellites are at the same longitude of ascending node. There are 9 passes made on the first day as opposed to only 8 for the case where both satellites are at the same longitude of ascending node. However there are only 6 passes made on the final day as opposed to 7 passes for the case where both satellites are the same node. The total number of passes made during the 30 day period is 309 passes in comparison to 316 passes for the case with both satellites at the same longitude of ascending node.

Table 4.14 summarizes the coverage properties for the various two satellite constellation designs.

**Table 4.14 Summary of Coverage Properties for Two Satellite Constellations**

| **Longitude of Ascending Node** | **Number of Daylight Passes** | **Minimum Passes Per Day** | **Maximum Passes Per Day** | **Average Passes Per Day** |
|---|---|---|---|---|
| Satellite 1 and 2: 72° | 316 | 7 | 12 | 11 |
| Satellite 1: 72°<br>Satellite 2: 108° | 307 | 6 | 12 | 10 |
| Satellite 1: 72°<br>Satellite 2: 36° | 309 | 6 | 12 | 10 |

### *4.7.3 Three Satellite Constellations*

The two-satellite constellation cases demonstrate the results for separating satellites by mean anomaly as well as by longitude of ascending node. For a three-satellite constellation, both options were again considered. The first design is to place all three satellites in an orbit with the same longitude of ascending node and spaced evenly by mean anomaly. The second design is to place the first satellite in an orbit with a longitude of ascending node of 36 degrees, the second satellite in an orbit with a longitude of ascending node of 72 degrees, and the third satellite in an orbit with a longitude of ascending node of 108 degrees. The three satellites were also spaced evenly by mean anomaly. Table 4.15 summarizes the coverage properties for each constellation.

**Table 4.15 Summary of Coverage Properties for Three Satellite Constellations**

| Longitude of Ascending Node | Number of Daylight Passes | Minimum Passes Per Day | Maximum Passes Per Day | Average Passes Per Day |
|---|---|---|---|---|
| Satellite 1, 2, and 3: 72° | 475 | 11 | 18 | 16 |
| Satellite 1: 36°<br>Satellite 2: 72°<br>Satellite 3: 108° | 446 | 10 | 18 | 15 |

### *4.7.4 Extended Operations*

The constellations described thus far have been designed for a period of 30 days. If operations are expected to last longer than a 30 day period, a different design may be necessary. Figure 4.25 shows the two satellite constellation with both satellites in orbit with the same longitude of ascending node extended out to 60 days.

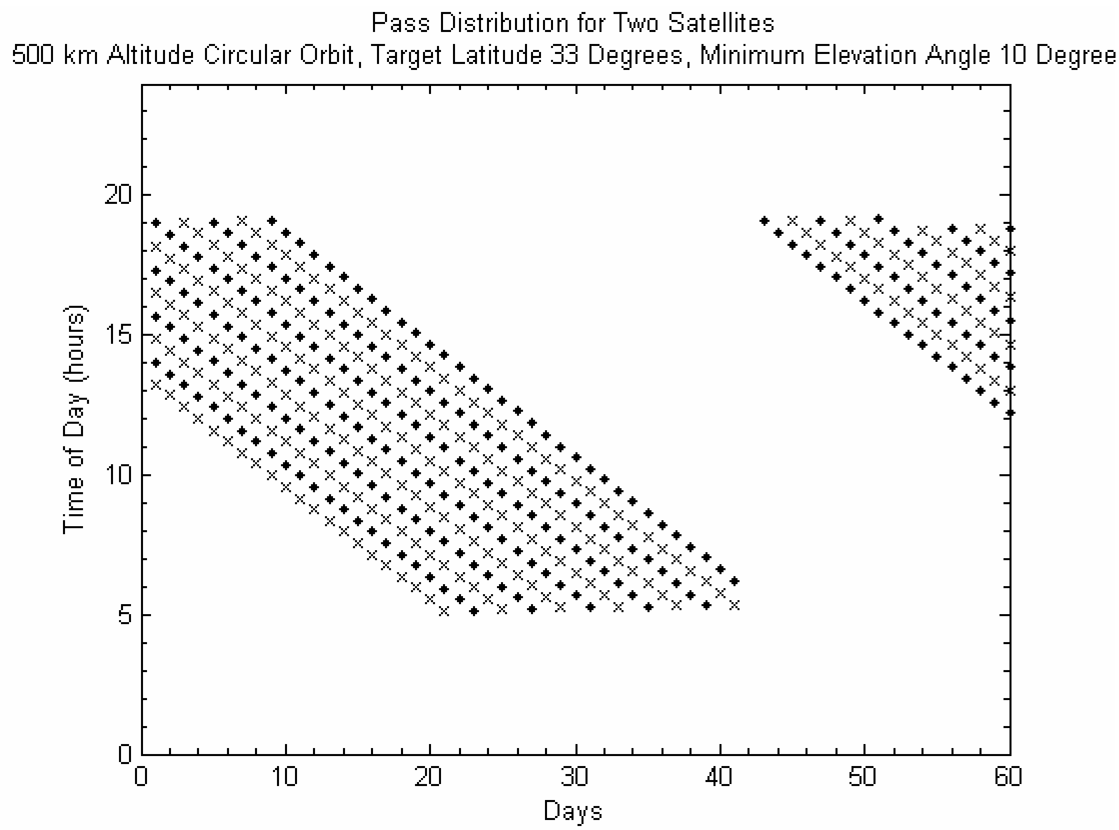


**Figure 4.25 Distribution of Passes for Two Satellites Over 60 Days**
**Satellite 1 and 2: 500 km Altitude Circular Orbit, 45° Inclination, 72° Longitude of Ascending Node, Minimum Elevation Angle 10°, Target Latitude 33°**

The results show that as the node continues to drift past the 30 day time period, the satellites make few to zero passes for a period of days. As the node drifts back toward its

original value the number of passes increases and the pattern begins again. Figures 4.26 and 4.27 show the other two satellite constellations extended out to a period of 60 days.

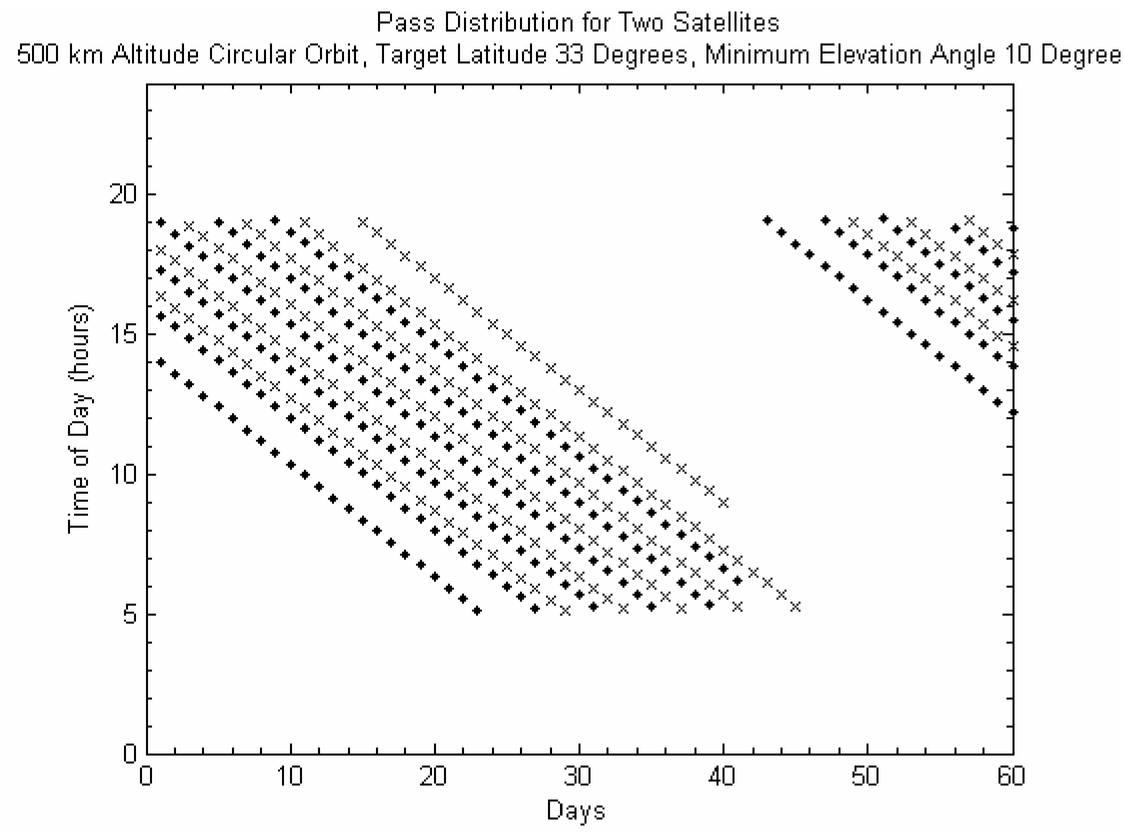


**Figure 4.26 Pass Distribution for Two Satellites Over 60 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Minimum Elevation Angle 10°**
**Satellite 1: 72° Longitude of Ascending Node**
**Satellite 2: 108° Longitude of Ascending Node**

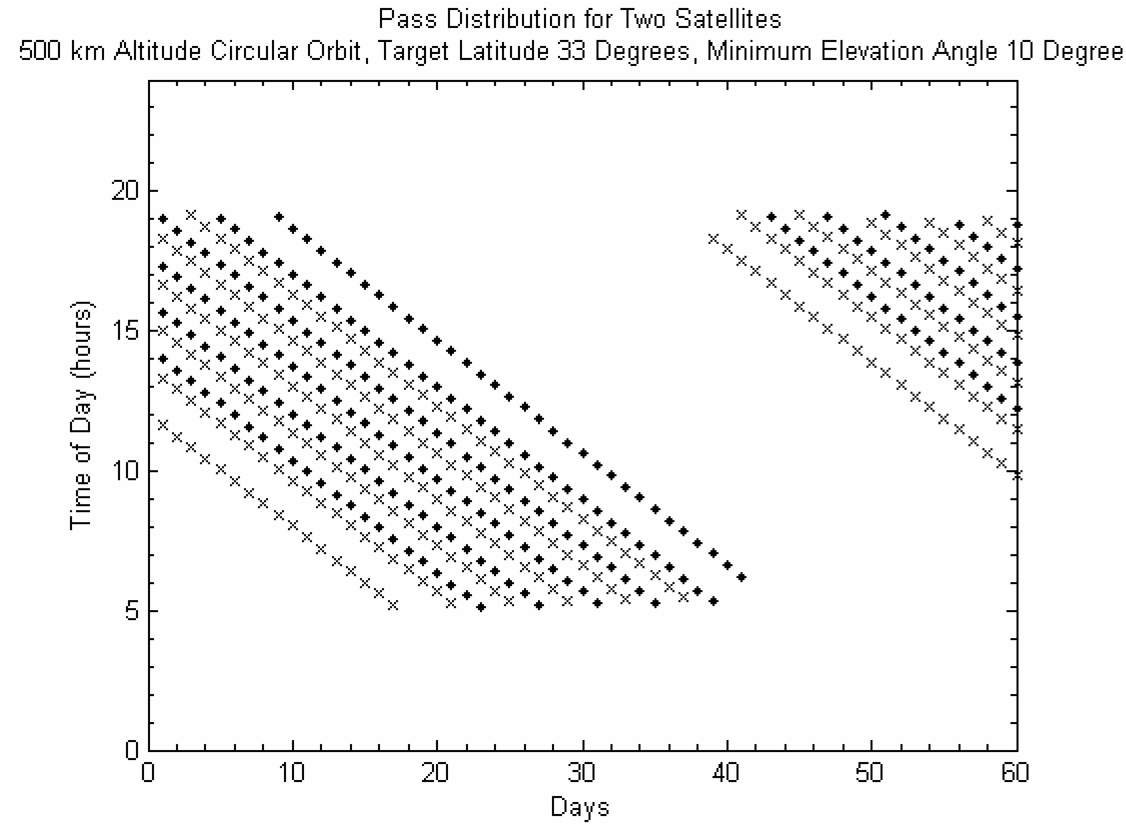


**Figure 4.27 Pass Distribution for Two Satellites Over 60 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Minimum Elevation Angle 10°**
**Satellite 1: 72° Longitude of Ascending Node**
**Satellite 2: 36° Longitude of Ascending Node**

The results for each case are similar. There is a period of several days where very few passes are made because the node has drifted to a value where most of the passes over the target are made during the night. Table 4.16 provides a summary of the coverage properties for each constellation over the 60 day period.

**Table 4.16 Summary of Coverage Properties for Two Satellite Constellations Over 60 Days**

| Longitude of Ascending Node | Number of Daylight Passes | Minimum Passes Per Day | Maximum Passes Per Day | Average Passes Per Day |
|---|---|---|---|---|
| Satellite 1 and 2: 72° | 448 | 0 | 12 | 7 |
| Satellite 1: 72°<br>Satellite 2: 108° | 449 | 1 | 12 | 7 |
| Satellite 1: 72°<br>Satellite 2: 36° | 471 | 2 | 12 | 8 |

As shown in figure 4.28 the pass distribution will continue in a periodic pattern as the node continuously drifts. If tactical operations are expected to last for an extended period of time, the periods of days with few to no passes might provide an undesirable coverage pattern.

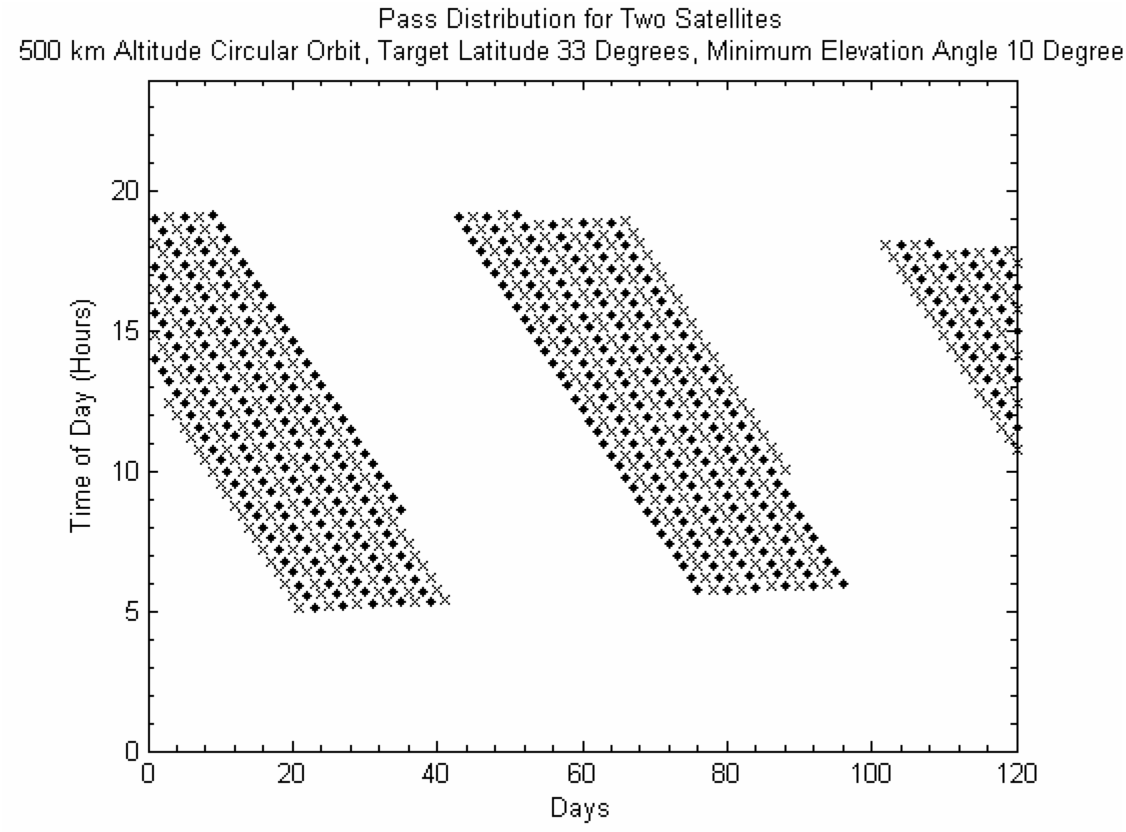


**Figure 4.28 Distribution of Passes for Two Satellites Over 120 Days**
**Satellite 1 and 2: 500 km Altitude Circular Orbit, 45° Inclination, 72° Longitude of Ascending Node, Minimum Elevation Angle 10°, Target Latitude 33°**

In order to avoid coverage gaps, a constellation can be designed to ensure satellite passes are made every day. By spacing the satellites evenly by longitude of ascending node, the coverage will not lapse during extended operations. Figure 4.29 shows a two satellite constellation with one satellite in an orbit with the longitude of ascending node at 72 degrees and the second satellite in an orbit with the longitude of ascending node at 252 degrees.

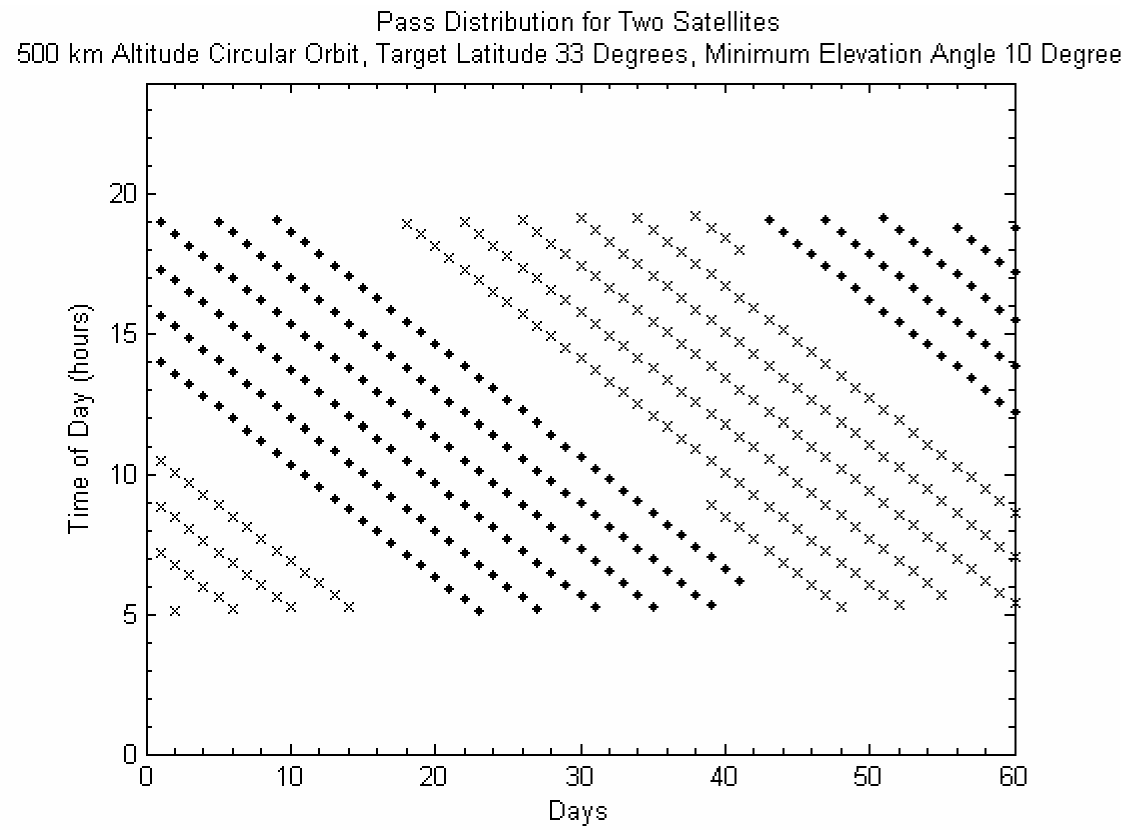


**Figure 4.29 Pass Distribution for Two Satellites Over 60 Days**
**500 km Altitude Circular Orbit, 45° Inclination, Minimum Elevation Angle 10°**
**Satellite 1: 72° Longitude of Ascending Node**
**Satellite 2: 252° Longitude of Ascending Node**

With the two orbits separated by 180 degrees of longitude of ascending node, the coverage is spread out so that there are no days with few or zero passes. Table 4.17 provides a summary of the coverage properties over the 60 day period.

**Table 4.17 Summary of Coverage Properties for Two Satellite Constellation Over 60 Days**

| Longitude of Ascending Node | Number of Daylight Passes | Minimum Passes Per Day | Maximum Passes Per Day | Average Passes Per Day |
|---|---|---|---|---|
| Satellite 1: 72°<br>Satellite 2: 252° | 439 | 6 | 9 | 7 |

# 5. Conclusions

## 5.1 Target Location

The latitude at which a target site is located plays an important role in determining the appropriate orbit for target coverage. If the target site is on the equator, an equatorial orbit should be used. In this case the orbit's inclination will match the target site's latitude. In an equatorial orbit, the satellite's field of view will always be over the equator and whenever the target site's longitude comes within the satellite's field of view, the target site will be visible to the satellite. No additional gains are made by increasing the orbital inclination above the equator. If the inclination is increased a few degrees but remains below the value of the Earth-central angle for a given altitude, the equator will still always be within the field of view of the satellite. However the average slant range from the satellite to the target will be increased which is undesirable for applications such as high resolution visible imaging. If the inclination is increased above the value of the Earth-central angle, the latitude of the target site will no longer always be within the field of view of the satellite and the number of daylight passes made will decrease.

If the target site is at a latitude that is above the equator but at a smaller value than the Earth-central angle, the case is slightly different. The altitude of the orbit will determine the Earth-central angle; for an altitude range of 200 to 800 km the Earth-central angles range from 14 to 27 degrees. Thus the latitude range that this case applies to will vary depending on the orbital altitude, but low latitudes such as 5 or 10 degrees will always fall in this category. If the satellite is placed in an equatorial orbit, the latitude of the target site will always be within the satellite's field of view because the

latitude value is less than the Earth-central angle. The satellite's inclination can be raised above the equator and the latitude of the target site still always be within the field of view. This will be the case if the inclination selected is less than the difference between the Earth-central angle and the target latitude. For example the 350 km altitude orbit has an Earth-central angle of about 18.6 degrees. For a target site at a latitude of ten degrees, the difference between the Earth-central angle and target latitude is about 8.6 degrees. At an inclination above 8.6 degrees, the target latitude would no longer always be within the field of view of the satellite which would mean a decrease in the number of passes. The results for the 350 km case showed a decrease in the number of passes at inclinations above 8 degrees. Since the number of passes does not vary significantly for the range of inclinations in which the target latitude is always within the flied of view of the satellite, a simple solution is to just choose an equatorial orbit. However the average slant range will be minimized at an inclination near the target latitude. In the inclination range where the latitude is always within the field of view, the average slant range might be higher at an equatorial orbit than an inclination closer to the target latitude. The tradeoffs between slant range and number of passes will have to be considered before choosing an orbit.

Another case is when the Earth-central angle is less than the value of the target latitude. Depending on the orbit altitude this range of latitude would begin around 18-27 degrees. Since most recent theater operations have occurred at latitudes above 27 degrees, this region is of high interest. There are two types of coverage that can be provided by low altitude orbits for targets in this latitude region. The first type of coverage is when a satellite makes one pass over the latitude of the target during each orbital period. This type of coverage will include the range of inclinations from the

latitude of the target up to a value near the Earth-central angle plus the latitude of the target. The second is when the satellite makes two passes, one as it is ascending and one as it is descending, over the latitude of the target. This range will include inclinations higher than the Earth-central angle plus the latitude of the target. The highest amount of daylight passes will be made by the first type of coverage and will occur at an inclination near the value of the latitude plus the Earth-central angle. The results have shown the second case to be undesirable because the number of daylight passes decreases significantly. The average slant range also decreases but not enough to compare with the first type of coverage. For the first type of coverage, as the inclination increases above the target latitude, the number of daylight passes increases but so does the average slant range. The tradeoff between slant range and passes is an important consideration for orbit selection.

### 5.2 Orbital Altitude

The altitude of the orbit is another important parameter that affects the target coverage a satellite provides. For a target site located on the equator and a satellite in an equatorial orbit, increasing the altitude will decrease the number of daylight passes made over the target. This trend corresponds with the increased period a higher altitude orbit will have. The period of the orbit is important because during the time it takes the satellite to complete an orbit, the Earth will rotate. For a higher period, there will be fewer successive passes because the Earth will rotate more during each orbit than for a shorter period and the target will be out of view after less passes than for a shorter period. Increasing the orbit altitude also increases the field of view and hence Earth-central angle

of the satellite but since the latitude of the target is always within view, a larger field of view does not add any increase in the number of passes. Increasing the altitude of the orbit also increases the average slant range to the target. Since the satellite is at a higher altitude, the distance from the satellite to the target will also be higher.

If the target site is at a latitude of ten degrees, the effects of increasing the orbit altitude are similar to the equatorial case. At a given altitude the number of daylight passes and average slant range vary with inclination, but there are still overall trends that are evident for varying altitudes. As the orbit altitude increases, the period of the orbit increases and the number of daylight passes decreases. The average slant range also increases as the orbit altitude increases.

For a target latitude of 33 degrees, orbit altitude has several important effects. At a particular altitude the number of daylight passes and average slant range will depend on inclination, but there are still general trends that can be observed for varying altitudes. As the altitude of an orbit is increased, the Earth-central angle of the satellite is increased. This corresponds to an increase in the number of successive passes that are made by a satellite. If the orbit altitude is increased, the number of daylight passes made increases. The average slant range also increases with increasing orbit altitude.

### 5.3 Simple Optimization for Maximum Number of Passes

For a given orbit altitude, the inclination can be optimized to provide the maximum number of daylight passes. For a target latitude at the equator the optimal inclination is zero as discussed previously. For a target latitude of ten degrees, an inclination of zero will also yield the maximum number of daylight passes. For cases

where the target latitude is greater than the Earth central angle, the optimal inclination will depend on the orbital altitude, target latitude, and other constraints placed on the optimization. Only this type of case will be discussed further.

### *5.3.1 Constrained Altitude, Unconstrained Elevation Angle and Slant Range*

For a target at a latitude of 33 degrees, the optimal inclination will be near the value of the Earth-central angle plus the target latitude. The Earth central angle for the given orbital altitude can be calculated using equation (3.1). A numerical search of inclination values near the Earth-central angle plus the target latitude will quickly yield the optimal inclination. This optimization scheme will work for any target latitude that is larger than the Earth central angle of the satellite altitude.

### *5.3.2 Constrained Altitude and Elevation Angle and Unconstrained Slant Range*

For a given orbit altitude and a minimum elevation angle, the orbital inclination can be optimized to provide the maximum number of daylight passes. The optimum inclination will be near the value of the effective Earth central angle plus the target latitude. The effective Earth central angle for the given minimum elevation angle and altitude can be calculated using equation (3.2). A numerical search of inclination values near the effective Earth central angle plus the target latitude can be used to find the optimal inclination.

### *5.3.3 Constrained Altitude and Constrained Slant Range*

For a given orbit altitude and maximum slant range, the orbital inclination can be optimized to provide the maximum number of daylight passes. The optimum inclination will be near the value of the effective Earth central angle plus the target latitude. The effective Earth central angle for the given altitude and slant range can be calculated using

equation (3.3) to (3.5). A numerical search of inclination values near the effective Earth central angle plus the target latitude will yield the optimum inclination value.

### *5.3.4 Unconstrained Altitude and Constrained Slant Range*

For a constrained slant range, the orbital inclination and altitude can be optimized to provide the maximum number of daylight passes. The optimum inclination for an altitude value will be near the value of the effective Earth central angle plus the target latitude. A numerical search over altitude values using the optimized inclination at each altitude will yield the optimum altitude value.

## 5.4 Elliptical Orbits

For the elliptical test case examined, the number of daylight passes made was not significantly improved over a circular orbit. In addition the slant range to the target was increased, an undesirable effect. For these reasons elliptical orbits were not further explored. However there may be some applications that benefit from the use of elliptical orbits. The test case showed a higher average pass length for the elliptical orbit in comparison to the circular orbit. Elliptical orbits offer the benefit of increased time spent over a target because a satellite will usually be over the target when it is near the apogee of its orbit and traveling at a slower velocity than a satellite in a circular orbit. For some applications the time spent over the target during a pass may be an important factor and the use of elliptical orbits might be considered.

## 5.5 Multi-Objective Optimization

For satellite missions such as high resolution visible imaging, satellite coverage properties must be evaluated using multiple criteria. The maximum number of imaging opportunities is desired but the slant range from the satellite to the target should also be minimized to allow for high resolution images. As the number of daylight passes increases, the slant range typically increases as well, creating a tradeoff between the two properties. In order to take into account both objectives, a weighted optimization algorithm can be used to select optimal orbit parameters. The algorithm developed allows a weight to be placed on the number of daylight passes made and on the average slant range in order to represent the relative importance of each property. The algorithm can be used to find orbits which provide a balance between the number of daylight passes made and the average slant range from the satellite to the target.

## 5.6 Satellite Constellations

Constellations of satellites can be designed using the optimal orbit properties determined for a single satellite. Keeping all the satellites of a constellation at the same orbital altitude and inclination is a common design technique because the satellites will not be affected differently by J2 orbital perturbations and the integrity of the constellation will be maintained.

The optimal orbit properties of a single satellite include initial orbit altitude, inclination, and longitude of ascending node. Additional Satellites can be placed in the same orbit but separated by mean anomaly or in another orbit separated by longitude of ascending node. Adding additional satellites will increase the number of daylight passes

made over the target.  Constellations can be designed to maximize coverage over a short period such as 30 days or to spread coverage out evenly for extended operations.

## 5.7 Recommendations for Future Work

The satellite propagation included the J2 perturbation which causes a regression of the node for an orbit.  The perturbation was included because it has an important impact on orbit coverage.  Another parameter that could be included is the drag force on a satellite.  Satellites in low Earth orbit experience a significant force due to drag which could be modeled in order to see its impact on target coverage.

The orbit optimization algorithm included two important coverage properties, the number of daylight passes and the average slant range.  There are various other coverage properties that could be included in an optimization algorithm.  The algorithm could include the objectives of maximizing the total coverage time over the target or minimizing the average or maximum time between passes. An algorithm could also be developed to optimize constellations of satellites.

The focus of this research effort was on orbits which will be used for satellites collecting visible imagery.  Tactical satellites may also serve other missions such as communications or types of surveillance other than visible imaging.  Since other applications may be able to operate at night, the requirement for daylight passes would not necessarily be included. Other requirements such as a minimum time between passes could be explored.

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## Vita

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| **REPORT DOCUMENTATION PAGE** | *Form Approved* *OMB No. 074-0188* |
|---|---|



| **1. REPORT DATE** *(DD-MM-YYYY)* | **2. REPORT TYPE** | **3. DATES COVERED** *(From – To)* |
|---|---|---|
| 22 Mar 07 | **Master's Thesis** | Aug 2005 – Mar 2007 |

**4. TITLE AND SUBTITLE**

Optimal Orbital Coverage of Theater Operations and Targets

**5a. CONTRACT NUMBER**

**5b. GRANT NUMBER**

**5c. PROGRAM ELEMENT NUMBER**

**6. AUTHOR(S)**

Sugrue, Kimberly, A., Captain, USAF

**5d. PROJECT NUMBER**

**5e. TASK NUMBER**

**5f. WORK UNIT NUMBER**

**7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)**

Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Way
WPAFB OH 45433-7765

**8. PERFORMING ORGANIZATION REPORT NUMBER**

AFIT/GA/ENY/07-M17

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

N/A

**10. SPONSOR/MONITOR'S ACRONYM(S)**

**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**

**12. DISTRIBUTION/AVAILABILITY STATEMENT**

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**

The use of satellites as a tactical asset to support theater operations is a desired capability for future space operations. Unlike traditional satellite systems designed to provide coverage over the entire globe or large regions, tactical satellites would provide coverage over a small region which can be modeled as a single ground point defined by a latitude and longitude. In order to provide sufficient utility as a theater asset, a satellite should be placed in an orbit that provides a maximum amount of coverage of the target ground point. This study examined the optimization of orbit parameters to maximize the number of passes made over a target. An optimization algorithm was developed to maximize the number of passes made while also minimizing the distance from the satellite to the target. Single satellite coverage properties as well as two and three satellite constellations were analyzed.

**15. SUBJECT TERMS**

Satellite Coverage, Orbit Coverage, Tactical Satellite, Theater Coverage, Target Coverage, Optimal Orbits

| **16. SECURITY CLASSIFICATION OF:** | | | **17. LIMITATION OF ABSTRACT** | **18. NUMBER OF PAGES** | **19a. NAME OF RESPONSIBLE PERSON** |
|---|---|---|---|---|---|
| REPORT | ABSTRACT | c. THIS PAGE | | | Lt Col Nathan A. Titus |
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